

# Internal Control System, Earnings Quality and the Dynamics of Financial Reporting\*

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## Abstract

Using an earnings management model in which managers can manipulate the earnings announcement when the firm's internal control system fails, this paper proposes a measure of earnings quality, based on Ganuza and Penalva's (2010) notion of integral precision that can be estimated using either the time series of earnings or the relation between earnings and prices. I show that large scale frauds are likely to be observed when the firms' internal control system is good. By contrast, it is the high frequency of small frauds that reveals a bad control system. I also find that earnings may unboundedly increase investors' uncertainty and that a better control system, though it leads to better earnings quality ex ante, also entails higher ex post uncertainty over the right tail of the distribution of earnings. In a dynamic extension in which managers are allowed to restate prior information, a natural link between the firm's balance sheet and the earnings announcement arises. In this setting, I examine the determinants of restatements and also the validity of some empirical measures of earnings quality. Neither the predictability nor the smoothness of earnings would be valid metrics of earnings quality.

*Keywords:* Earnings management, Restatements, Structural estimation.

*JEL Classification:* D82 (Asymmetric and Private Information), D83 (Search; Learning; Information and Knowledge), D84 (Expectations; Speculations).

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# 1 Introduction

Financial reporting is a game between investors and firms' management in which managers try to exploit the discretion in GAAP to influence investors' interpretation of their firms' financial conditions while investors seek to uncover firms' "true" financial conditions from the firms' reports. There are many components to this financial reporting game, including the specification of GAAP, the integrity of management, the nature of the transactions firms engage in, and the incentives of management.

The purpose of the present paper is study how the effectiveness of firms' internal control systems ("ICS") affects the interplay between managers and investors when this financial reporting game unfolds over time. A firm's ICS in the model we study fluctuates: sometimes it is strong and sometimes it is weak. When the ICS is strong, managers have no alternative but to report their firms' underlying financial condition truthfully, whereas when it is weak, managers have considerable discretion in how they report their firms' financial condition. Managers know whether their firms' ICS is strong or weak at the time they make their reporting choices. Investors, by contrast, are resigned to make inferences about firms' ICS based on the reports firms release.

While, as noted at the outset, the quality of a firm's ICS is only one of several factors that influence the financial reporting game, it provides the basis for studying a variety of issues fundamental to financial reporting, including: (1) how to measure earnings quality from the time series of reported earnings, (2) the incentives of managers to issue restatements, (3) how book-to-market ratios influence firms' financial reporting and (4) how firms' financial reporting evolves over time. In the remainder of this introduction, I give an overview of how the study of this financial reporting game helps analyze each of these issues.

First, consider the issue of earnings quality. In the reporting game I study, one natural basis for measuring a firm's earnings quality involves using the probability that the firm's ICS is strong: the reports of a firm whose ICS is more reliable (i.e., more likely to be strong) could be considered of higher quality than those of a firm whose ICS is less reliable. Unfortunately, a more reliable ICS does not imply higher informativeness according to Blackwell's (1951) classical notion of informativeness. Even though a more reliable ICS implies a higher probability of a truthful report it also leads to more

aggressive reporting behavior when the firm's control system is weak. The latter effect reduces the informativeness of the firm's reports over the right tail of the distribution of reports, which explains why a more reliable ICS does not mean higher earnings quality in the Blackwell's (1951) sense.

Yet, this difficulty can be overcome by appealing to a new criterion of informativeness, known as *integral precision*, that was recently developed by Ganuza and Penalva (2010). I prove that ranking reporting systems by the probability the firms' ICS is effective is equivalent to ranking them by integral precision. I also show that the rankings generated by some traditional measures of earnings quality, such as the volatility of prices around earnings announcements and the persistence of reported earnings, are consistent with the rankings by integral precision, whereas the rankings generated by other traditional measures, such as the *smoothness* or *predictability* of earnings, are not consistent with integral precision. Because integral precision is consistent with well-founded ideas of economic usefulness (and in particular with Blackwell's sufficiency criterion) this provides a basis for resolving conflicts about which measures of earnings quality are preferred.

Second, consider the issue of financial restatements. In practice, restatements are often highly scrutinized actions. Sometimes they are initiated by an external party (e.g., the SEC or an independent auditor) and sometimes by the firm. But regardless of who initiates them, investors typically expect firms to sometimes restate prior reports, especially if investors are skeptical about the quality of the firms' ICS or have doubts about the reliability of the firm's balance sheet. In fact, when the firm's balance sheet exhibits low levels of credibility, the absence of a restatement could be perceived by investors as further evidence of manipulation. In turn, the manager's awareness of investors' skepticism provide incentives for the manager to engage in "strategic" restatements as a means to manage the overall credibility of the firm's balance sheet.

To understand managers' incentives to issue restatements I first treat restatements as part of the reporting game, namely I assume that, when the firm's ICS is weak, restatements can be manipulated to affect investors' perceptions, and, when the firm's ICS is strong, truthful restatements are automatically announced to correct any prior misstatements. In this context, I find that restatements convey no information beyond the information that is already present in the firm's book value because, for a given

level of the book value, the restating behavior of the firm under strong and weak ICS are identical. This is an equilibrium requirement: the manager realizes that maximizing investors' perceptions about the value of the firm requires maximizing the credibility of the firm's book value which in turn can only be attained if the restating behavior of the firm under weak and strong ICS are indistinguishable from one another.

When managers have no discretion over restatements and restatements only occur under a strong ICS (being thus truthful) the manager's strategic behavior is still strongly affected by the possibility of restatements. For example, a firm that has systematically reported good performance in the past and whose book value has grown very steeply over time may end up raising investor's skepticism. Investors may then interpret the absence of restatements in the future as further evidence of manipulation. The manager in turn may feel himself forced to report losses so as to raise the overall credibility of the firm's balance sheet. This tendency to reverse large prior reports is interesting because it shows that the lack of persistence of reported earnings may be due to problems in the firm's reporting system and, more generally, that the time series properties of reported earnings may strongly differ from the properties of the true earnings, especially when firms' control systems are unreliable.

Third, consider how the balance sheet and the income statement interact in this reporting game. Apart from the obvious mechanical interaction (earnings are sometimes retained thereby becoming part of the firm's book value) there is a more subtle one: in the model, the manager's incentives to engage in a strong manipulation of the firm's income statement depend on the size of the book value, and in particular, on the level of the book-to-market ratio. Larger reports typically lead to larger but also to less credible book values. In this context, the book-to-market ratio arises naturally as a measure of the credibility of firms' balance sheets and becomes a key determinant of managers' incentives to aggressively manipulate income statements. From an empirical point of view this result suggests that the book-to-market ratio may be not only a predictor of restatements but also a determinant of discretionary accruals.

Fourth, I study how the financial reporting game evolves over time. In particular I examine whether younger firms tend to be more aggressive than older firms in manipulating the income statement. Younger firms are indeed more aggressive than older firms are for two reasons. First, at the start of the firm's operation the level of infor-

mation asymmetry (about both the prospects of the firm and the characteristics of the firm's reporting system) between the manager and investors tends to be larger than in later periods, because investors learn over time. Managers tend to exploit investors' greater "ignorance" by manipulating the income statement in a more aggressive manner. Second, older firms tend to have larger book values but also face more serious credibility problems. As mentioned earlier, these greater credibility problems typically moderate the manager's tendency to aggressively manipulate the firm's reports.

## 1.1 Literature review

This paper builds on the earnings management literature (see e.g., Dye, 1988; Evans and Sridhar, 1996; Fischer and Verrecchia, 2000; and Guttman et al., 2007). I modify Dye (1988) by assuming that with positive probability the manager bears no cost from misreporting the firm's earnings. Also I assume that earnings accumulate over time and that the manager may revise or restate prior earnings reports. This assumption allows us to study links between the firm's balance sheet and the firm's reported earnings.

The question of what constitutes the quality of information is classical and has been studied in accounting (see e.g. Demski, 1973; Verrecchia, 1990; Dye & Sridhar, 2004), economics (see e.g., Athey and Levin, 2001; Ganuza and Penalva, 2010; Persico 2000) and statistics (see e.g., Lehmann, 1988) particularly since Blackwell (1951, 1953). The empirical literature on earnings quality measures is vast (few examples are Beaver, 1968; Collins and Kothari, 1989; Dechow and Dichev, 2002; Dechow, Ge and Schrand, 2010; Francis, LaFond, Olson and Schipper 2008; Jones, 1991 and Schipper and Vincent, 2003). An alternative theoretical assessment of accounting measures of earnings quality can be found in Ewert and Wagenhofer (2010, hereafter EW). EW is a two period earnings management model based on Fischer and Verrecchia (2000). In EW, the manager's objective is to both maximize the firm's stock price and smooth earnings. The manager is privately informed about some value relevant information but the prevailing accounting standard prohibits him from reporting that information in the first period. If the manager does misreport in the first period, he must reverse the misstatement in the second period (as in Evans and Sridhar, 1996 or in the two period model in Dye, 1988). Moreover, by violating the accounting standard, the manager experiences a quadratic cost. EW study the linear rational expectations equilibrium of this model

and find that the two metrics that seem to better reflect the idea of earnings quality are value relevance and the persistence of earnings. By contrast, in their model, the accrual metric (see e.g., Dechow and Dichev, 2002 or Jones, 1991) show an ambiguous relation with earnings quality.

I borrow from the cheap talk literature originated by Crawford and Sobel (1982). Newman and Samsing (1993) and Gigler (1994) provide two applications in accounting. They show how the presence of two types of audiences (investors in the capital market and potential entrants in the product market) determines the credibility of financial reports. By contrast, in this paper the mechanism that sustains the credibility of reports is given by investors being uncertain about whether the firm's ICS is effective.

Stocken (2000) also studies the credibility of financial reports. Appealing to a folk theorem, in a repeated game setting with imperfect monitoring, Stocken shows that under certain conditions the credibility of financial reports can almost always be ensured by the implicit threat of losing access to capital markets. By contrast, in the present paper, managers are concerned with the short term value of the stock price and thus the future provides no incentive for truth-telling. Yet, as in Stocken (2000), the communication is possible *ad infinitum*, even when the firm's ICS remains constant across time periods.

This paper is also related to the literature on psychological games (see e.g., Geanakoplos, Pearce and Stacchetti, 1989; and Brandenburger and Polak, 1996) and to cheap talk communication games with uncertainty about the sender's type, such as Sobel's (1985) theory of credibility and Benabou and Laroque's (1992) model of insider trading. To the best of our knowledge, this is the first paper that considers a continuous signal/message space and that provides a structural model for estimating the credibility of the sender. I expect the paper's results to have applications not only in the evaluation of earnings reports but also in other accounting settings where strategic communication is important, such as those involving financial analysts, forecasters, etc.

The dynamics of financial reporting is also studied by Einhorn and Ziv (2007). They consider a disclosure model –in which disclosures are costly as in Verrecchia (1983) and the manager's endowment of information is uncertain as in Dye (1985)– in which the likelihood that the manager receives some private information is serially correlated. They show that this inter-temporal linkage increases managers' propensity to withhold

information because disclosing information results in an implicit commitment to disclose information in the future.

Evans and Sridhar (1996) study how the uncertainty about the firm's internal control system affects the choice of financial reporting system in an agency setting. They show that an accrual system that tolerates some manipulation is generally desirable. Beyer and Sridhar (2005) study how the uncertainty about auditor's control system affect the auditor's choice of reporting system and the likelihood of audit failures.

The rest of the paper proceeds as follows. Section 2 presents the baseline model. Section 3 characterizes the properties of the equilibrium and discusses three issues: earnings quality, the magnitude of frauds and the location of kinks in the distribution of reported earnings. In Section 4 I consider the multi-period version of the model in which managers are allowed to restate information. In this setting, I examine the dynamics of financial reporting. Section 5 concludes.

## 2 The baseline model

In this section I study a single firm/one period model of earnings management in which a manager reports the firm's earnings to a group of competitive and risk neutral investors who are uncertain about whether the firm's ICS is effective. There are no restatements in the baseline setting. I relax this assumption in Section 4.

During the period, the following three events take place. First, the manager privately observes two mutually independent random variables: (i) the firm's true earnings  $\tilde{x}$ —whose p.d.f. and c.d.f. I denote by  $f(\cdot)$  and  $F(\cdot)$ —are normally distributed with mean  $\mu$  and variance  $\sigma^2$  and (ii) the state of the firm's internal control system which is represented by a binary random variable  $\tilde{\tau}$  indicating whether or not the ICS is effective.<sup>1</sup> With probability  $\gamma$ , the firm's ICS is strong ( $\tau = s$ ), which means that the manager must report the firm's true earnings. With probability  $(1 - \gamma)$  the firm's ICS is weak ( $\tau = w$ ), which means that the manager can manipulate the firm's reported earnings at no cost. I refer to  $\gamma$  as the firm's ICS quality. The determinants of  $\gamma$  include any institutional mechanism that limits the manager's ability to manipulate the firm's

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<sup>1</sup>Assuming that earnings are normally distributed simplifies the exposition but, for most of the analysis below, is not required; only the assumption that the distribution of earnings admits a continuous p.d.f. is required.

earnings reports. If the firm's ICS is weak, the manager bears no cost from misreporting the firm's earnings (see e.g., Crawford and Sobel, 1982; Gigler, 1994; Newman and Samsing, 1993; and Stocken, 2000).<sup>2</sup>

Second, after privately learning the state of the firm's ICS the manager reports the firm's earnings. Under a weak ICS, the manager manipulates the report so as to maximize the firm's stock price  $P$ .<sup>3</sup> I denote the firm's reporting system by the random variable  $\tilde{r}$  and its realization by  $r$ . Subsequently,  $r$  is referred to as the firm's report or as the firm's reported earnings.

Third, risk neutral and Bayesian investors set the price of the firm's shares at their expected value using all publicly available information. The structure of the game is common knowledge.

Three points are worth noting here. First, even though I assume that earnings are never directly verified by investors, the analysis would not be altered if along with the manager's report investors had access to a public and noisy signal of the underlying true earnings. Second, the analysis could apply to settings where the manager's report consists of several items but the manager has discretion in reporting only some of those items. Third, to simplify the exposition, the model assumes only one firm whose ICS quality is known to investors. This can be extended to a setting with a continuum of firms whose ICS quality is uncertain. In such setting,  $\gamma$  would denote the average ICS quality of the population of firms operating in the market. I will exploit this reinterpretation in the discussions below. I will also assume this kind of heterogeneity when estimating the distribution of ICS quality across U.S. firms in Section 3.2.1.

### 3 The single-period reporting game

In this section I solve the baseline model, and in Section 4 I show how the static game naturally extends to the multi-period case.

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<sup>2</sup>This cheap-talk assumption might seem strong when applied to earnings announcements. Apart from tractability considerations, we make this assumption to stress that investors' disbelief is sufficient both to impose discipline on the manager's behavior in a given period and to induce the kind of reversal mechanism on accruals that is often described in the literature (see e.g. Melumad and Kirschenheiter 2004).

<sup>3</sup>One way of motivating the manager's objective of maximizing the firm's current stock price is to assume an overlapping generations model where different generations of short lived managers overlap (as in Dye, 1988).



A reporting strategy  $\varphi(\cdot)$  is a probability density function over possible reports the manager makes when the firm's ICS is weak.<sup>4</sup> When the firm's ICS is strong, the manager has no discretion over reports, hence  $\tilde{r} = \tilde{x}$ . An equilibrium, as described next, specifies the relationship between a reporting strategy and a pricing function.<sup>5</sup>

**Definition 1** *A Perfect Bayesian Nash (PBN) equilibrium consists of a reporting strategy  $\varphi(\cdot)$  and a pricing function  $P(\cdot)$  such that: (i) given  $P(\cdot)$ , the manager's reporting strategy  $\varphi(\cdot)$  is such that for any  $r \in \text{support}\{\varphi(\cdot)\}$ , then  $r \in \arg \max_{\hat{r}} P(\hat{r})$ , and (ii) given  $\varphi(\cdot)$ , the pricing function is defined as  $P(r) \equiv E(\tilde{x} | \tilde{r} = r)$ .*

Thus, in equilibrium the manager chooses a reporting strategy that maximizes the firm's stock price, and investors update the firm's stock price upon learning the manager's report using Bayes rule. In particular, investors know that the firm's reporting system  $\tilde{r}$  is a mixture of two distributions: the distribution of the firm's true earnings  $f(\cdot)$ , and the distribution of the manipulated earnings  $\varphi(\cdot)$  chosen by the manager to maximize the firm's stock price when the ICS is weak.<sup>6</sup>

This definition of equilibrium implicitly rules out the possibility that the manager conditions his reporting strategy on the realization of the true earnings  $x$ . This restriction is natural because  $x$  is payoff irrelevant for the manager since he experiences no cost from misreporting earnings when the ICS is weak. Moreover, one can show that this restriction is without loss of generality.<sup>7</sup>

The next proposition asserts the existence of a unique equilibrium.

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<sup>4</sup>For simplicity, I define reporting strategies in terms of probability densities rather than distributions, which is more standard. This might seem restrictive because it rules out two possibilities: (i) the manager's reporting strategy could be a discontinuous distribution (ii) even if the manager's reporting strategy is a continuous distribution, it might not have a p.d.f. But both (i) and (ii) can be ruled out as part of an equilibrium when the distribution of earnings is absolutely continuous.

<sup>5</sup>Recall, for this definition, that the support of a continuous random variable that has a density is the set over which its density is positive.

<sup>6</sup>As Aumann (1964) has noted, when the "type" space is uncountable, modeling mixed strategies as maps from types to distributions over pure strategies is not well defined. To define a mixed strategy properly a randomizing device must be introduced. Yet this is not a problem in our setting because even though the manager privately observes a continuous random variable his type is binary (weak or strong).

<sup>7</sup>In any equilibrium, the pricing function coincides with the one that arises in equilibrium when one restricts attention to the class of reporting strategies that are independent of  $x$ . There are equilibria in which the manager conditions his report on his private observation of the firm's true earnings  $x$ . For instance one can easily construct equilibria in which the manager's reporting strategy is decreasing in  $x$ . However, we have not been able to rule out the existence of equilibria in which the manager's reporting strategy is positively related to the true earnings  $x$ .

**Proposition 1** *There exists a unique PBN equilibrium characterized as follows: (i) the manager's equilibrium reporting strategy is given by*

$$\varphi(r) = \begin{cases} \frac{\gamma}{1-\gamma} \frac{r-c}{c-\mu} f(r) & \text{if } r \in [c, \infty) \\ 0 & \text{if } r \notin [c, \infty) \end{cases} . \quad (1)$$

where  $c = c(\gamma)$  is defined by

$$\int_c^\infty \varphi(r) dr = 1. \quad (2)$$

(ii) *The pricing function is given by*

$$P(r) = \min(r, c).$$

I sketch the proof of Proposition 1 by postulating (and then verifying) that any equilibrium reporting strategy must satisfy the following property.

**Property 1** *The support of  $\varphi(\cdot)$  is a right-tailed interval, namely there exists a real number  $c$ , such that  $\varphi(r) > 0$  if and only if  $r \geq c$ . I refer to the set  $[c, \infty)$  as the right tail of the distribution of earnings.*

An immediate implication of this posited property is that the pricing function must fully impound the manager's report over the left tail ( $r < c$ ) because any report in that region fully reveals that the firm's ICS worked ( $\tau = s$ ), which in turn means that the report was truthful. A second implication of this property is that the pricing function  $P(\cdot)$  must be both maximal and constant over the set  $[c, \infty)$ . Otherwise,  $\varphi(\cdot)$  would not be the manager's optimal strategy when he can manipulate his report, i.e., when  $\tau = w$ . Combining these two observations and assuming that the pricing function is continuous, then  $P(r)$  must satisfy  $P(r) = c$  for all  $r \in [c, \infty)$ .<sup>8</sup> In summary, the equilibrium pricing function must be given by

$$P(r) = \begin{cases} r & \text{if } r < c \\ c & \text{if } r \geq c \end{cases} . \quad (3)$$

We will see that the above characterization of the pricing function along with Bayes rule completely determines the manager's equilibrium reporting strategy  $\varphi(\cdot)$ . In fact,

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<sup>8</sup>The pricing function must be continuous. It is clearly continuous over the left tail. It must also be continuous over the right tail. The question is whether it can be discontinuous at  $c$ . It is easy to show that such discontinuity cannot arise: informally that would require that the manager understates the true earnings when the firm's ICS is weak.

for the above description of the pricing function to be consistent with the Bayesian rationality of investors, the manager's equilibrium report  $\tilde{r}$  must induce constant expectations over the right tail, i.e., in equilibrium, investors' expectations must be flat over this right tail:

$$E[\tilde{x}|\tilde{r} = r] = c \text{ for all } r \in [c, \infty),$$

which is equivalent to

$$\Pr(\tilde{\tau} = s|r) r + \Pr(\tilde{\tau} = w|r) \mu = c \text{ for all } r \in [c, \infty). \quad (4)$$

This equation uses the fact that the firm's report is fully informative about  $\tilde{x}$  when its ICS is strong and is completely uninformative when its ICS is weak. Defining the level of credibility of a report as  $\Pr(\tilde{\tau} = s|r)$ , i.e., the posterior probability that the firm's ICS is effective, equation (4) shows that, in equilibrium, higher reports must have a lower level of credibility. By Bayes' rule, the report's credibility is given by:

$$\gamma(r) \equiv \Pr(\tilde{\tau} = s|r) = \frac{\gamma f(r)}{\gamma f(r) + (1 - \gamma) \varphi(r)}. \quad (5)$$

Substituting equation (5) into equation (4) and solving for  $\varphi(r)$  we get our candidate equilibrium reporting strategy:

$$\varphi(r) = \frac{\gamma}{1 - \gamma} \frac{r - c}{c - \mu} f(r), \quad (6)$$

In the Appendix, I show that  $c$  is uniquely determined as that value such that the following conditions hold:

$$\int_c^\infty \varphi(r) dr = 1 \text{ and } \varphi(r) \geq 0 \text{ for all } r. \quad (7)$$

(Even though Proposition 1 is stated for the specific case of normally distributed earnings, the existence and uniqueness of a solution to equation (7) is general and only requires that the distribution of earnings is absolutely continuous.)

Notice that the equilibrium reporting strategy is not deterministic. The non-existence of an equilibrium in pure strategies is attributable to the fact that if the manager wishes to maximize the firm's market value when the ICS is ineffective, he cannot misreport earnings in a predictable fashion. For example, if he reported the highest possible earnings, investors would interpret this as a sure sign of the breakdown in the firm's ICS

and would, therefore, disregard the report; no price reaction to the report would then take place.<sup>9</sup>

### 3.1 Earnings quality

This section uses the reporting model developed in the preceding section as a context for studying how to evaluate measures of earnings quality. Besides developing new ex-ante criteria by which to evaluate earnings quality, the section stresses the distinction between ex-ante measures of earnings quality (before any reports are realized) and ex-post notions of earnings quality. The main contribution of this section is to provide a new measure of earnings quality that has strong theoretical foundations and can be readily estimated using the time series properties of reported earnings.

As was noted in the Introduction, the early literature in decision theory (e.g., Blackwell, 1951) proposed a criterion to rank information systems (or signals) known as Blackwell's sufficiency criterion.<sup>10</sup> Blackwell's is still the most fundamental criterion of informativeness. However, it has been considered as excessively restrictive (see e.g., Lehmann, 1988).<sup>11</sup> More recent research in economics has developed alternative criteria to rank information systems in less restrictive ways than Blackwell's criterion (see e.g., Athey & Levin, 2001; Ganuza and Penalva 2010). In this section, I employ the results of this recent literature. In particular, I adopt Ganuza and Penalva's (2010) notion of *integral precision* as a criterion of informativeness. Informally, integral precision is an ordering of signals based on the property that more informative signals lead to greater variability of conditional expectations.

The formal definition of integral precision is presented next.<sup>12</sup>

**Definition 2 (IP)** *Given two signals  $\tilde{r}_1$  and  $\tilde{r}_2$  about a third random variable  $\tilde{x}$ , the random variable  $\tilde{r}_1$  is said to be more integral precise than the random variable  $\tilde{r}_2$  if  $\tilde{Y} \equiv E[\tilde{x}|\tilde{r}_1]$  is*

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<sup>9</sup>The non existence of pure strategy equilibria is also a consequence of the continuous signal space we are using. Existence of pure strategy equilibria would be recovered in a discrete state/signal space (see e.g., Benabou & Laroque, 1992).

<sup>10</sup>Roughly, according to Blackwell's sufficiency criterion, an information system A is more informative than another information system B, if all possible decision makers prefer information system A to information system B.

<sup>11</sup>Persico (2000) notes that: "there are very few pairs of signals that are ranked in terms of sufficiency, including some that cannot be ranked despite one signal appearing intuitively to be more informative than the other".

<sup>12</sup>For more details, see Shaked and Shantikumar (2007) or Ganuza and Penalva (2010).

greater than  $\tilde{Z} \equiv E[\tilde{x}|\tilde{r}_2]$  in the convex order. That is,  $E[\psi(\tilde{Y})] \geq E[\psi(\tilde{Z})]$  for all convex valued functions  $\psi(\cdot)$  (when both of these expectations exists).

The more the signal  $\tilde{r}$  is able to move expectations away from  $\tilde{x}$ 's priors, the more informative the signal is about  $\tilde{x}$ . Integral precision is consistent with standard notions of information including Blackwell's (1951) sufficiency criterion. If  $\tilde{r}_\gamma$  is more informative than  $\tilde{r}_{\gamma'}$  in the Blackwell's sense, then  $\tilde{r}_\gamma$  is more integral precise than  $\tilde{r}_{\gamma'}$  (see Theorem 1 (i) in Ganuza and Penalva, 2010). The converse is of course not true.

The notion of earnings quality I use in the sequel is that of integral precision.

**Definition 3 (EQ)** *Given two earnings reporting systems  $\tilde{r}_1$  and  $\tilde{r}_2$  about the firm's true earnings  $\tilde{x}$ , I say that  $\tilde{r}_1$  has higher earnings quality than  $\tilde{r}_2$ , if and only if  $\tilde{r}_1$  is more integral precise than  $\tilde{r}_2$ .*

One benefit of this definition of earnings quality is that it does not depend on the preferences of decision makers or the decision context.

The next corollary shows how earnings quality EQ can be operationalized in our setting.

**Corollary 1** *In equilibrium, an increase in  $\gamma$  always leads to higher EQ. Therefore, EQ can be indexed by  $\gamma$ .*

A better ICS, that is a higher  $\gamma$ , induces greater integral precision. Thus, earnings quality EQ can be identified with the quality of the firm's ICS. In fact, the ordering by  $\gamma$  not only coincides with the ordering by integral precision but also with two other notions of informativeness: the monotone information order (MIO) due to Athey and Levin (2001) and the related concept of effectiveness due to Lehmann (1988).<sup>13</sup>

EQ measures the information quality ex-ante - i.e., before an earnings report is released - and is, thus, a useful metric for predicting the informational quality of future

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<sup>13</sup>MIO is defined as follows. Given a prior  $F \in \Delta(\Omega)$ , suppose  $X, Y$  are signals leading to posterior beliefs  $F(|X), G(|Y)$  that satisfy FOSD, then  $G \succ_{MIO} F$  if for all  $q \in [0, 1]$ ,

$$G(\cdot|G_Y(Y) \geq q) \succ_{FOSD} F(\cdot|F_X(X) \geq q) \quad (8)$$

Effectiveness is defined as follows. Suppose  $X, Y$  are signals leading to posterior beliefs  $F(|X), G(|Y)$  that have the monotone likelihood-ratio property. Then  $G$  is more effective than  $F$ ,  $G \succ_L F$ , if  $G^{-1}(F(x|\omega)|\omega)$  is increasing in  $\omega$ .

reports. Yet, better information ex-ante does not generally mean better information ex-post. In other words, although knowledge of  $\gamma$  would help investors to predict the informational quality of future reports,  $\gamma$  alone may be of little help to investors in assessing the reliability of past reports, for example, the extent to which the firm's balance sheet is accurate. To see this point, assume that the firm's ICS is very good, i.e.,  $\gamma$  is close to one. Investors would then assess the firm as having high earnings quality  $EQ$  and would therefore expect very informative reports in the future. Yet, if the manager reported an extremely large number in the past, investors would presumably be very uncertain about the reliability of that report. The next two results show that this is in fact the case in equilibrium.

One way to measure the ex-post informational quality of a report is by the posterior probability that the firm's ICS worked, as that probability determines the credibility of the report, i.e., the extent to which investors perceive the report was drawn from an informative experiment.

**Corollary 2** *The posterior probability that the ICS worked ( $\Pr\{\tau = s|r\}$ ) decreases in the report's magnitude and is given by*

$$\gamma(r) = \begin{cases} \frac{c-\mu}{r-\mu} & \text{if } r < c \\ 1 & \text{if } r \geq c \end{cases} . \quad (9)$$

Corollary 2 says that the probability that the report is perceived as informative ex-post (which I refer to as the report's credibility) is monotonically decreasing in its magnitude. Ex post, the informational content of an earnings report depends on its magnitude. This idea, although intuitive, is not present in many empirical measures of earnings quality.<sup>14</sup>

Corollary 2 is the key to understanding the next result: an earnings report may increase investors' uncertainty about the firm's true earnings. In fact, the firm's report may unboundedly increase investors' uncertainty. To establish this result, I consider how the earnings report  $\tilde{r}$  affects investors' residual uncertainty, represented by the variance of the firm's earnings conditional on the report.<sup>15</sup>

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<sup>14</sup>Neither some accrual based measures such as the Dechow Dichev (2002); nor the accounting based measures such as the predictability, persistence or smoothness of earnings; nor the market based measures, such as the ERC and its variations, capture the idea that the average magnitude of reports may have to do with the informational quality of the reports.

<sup>15</sup>Since we assume risk neutrality, this type of uncertainty has no impact on market prices but it is

**Proposition 2** (i) For all  $r > c$  an increase in  $r$  induces a mean preserving spread in the posterior distribution of un-managed earnings  $\tilde{x}|_{\tilde{r}=r}$ , i.e.,  $\frac{\partial}{\partial r} \int_{-\infty}^y \Pr(\tilde{x} > u|r) du \leq 0$  for all  $y$ .  
(ii) Furthermore,  $\lim_{r \rightarrow \infty} \text{var}(\tilde{x}|r) = \infty$ .

The first part of Proposition 2 is intuitive: investors' residual uncertainty increases in the magnitude of the announcement. This result is explained by the fact that in equilibrium larger reports are relatively less credible. Proposition 2 (ii) shows the strength of this effect: reports may unboundedly increase investors' uncertainty. This is perhaps surprising because one would typically conjecture that investors' uncertainty about the firm's true earnings cannot exceed their uncertainty prior to learning the report as, after all, rational investors can always disregard any information they consider unreliable. That conjecture is true if the firm's ICS is always weak. But if, with some positive probability the firm's ICS is strong, things are different. The firm's report will increase the spread of the posterior distribution of earnings (as perceived by investors) when the report exceeds a certain threshold. After the firm releases a very high report, investors weigh two extreme possibilities: the firm's ICS is strong and the firm's value is truly very high or else the firm is engaging in a major overstatement of its true earnings. The larger the report the more investors get torn between these two alternative explanations for the high reported earnings. As Proposition 2 shows, investors' uncertainty about the firm's value grows unboundedly as  $r \rightarrow \infty$  while (perhaps paradoxically) investors become almost sure that the report is fraudulent and uninformative (note that  $\lim_{r \rightarrow \infty} \gamma(r) = 0$ ).

Proposition 2 and the related idea that ex-ante and ex-post notions of earnings quality can differ stands in stark contrast with the results of standard rational expectations literature, where ex-ante and ex-post notions of any sort of information quality coincide. The reason for the latter is that in these standard models the residual uncertainty (obtained after a firm's report is distributed), and more generally the information quality of the accounting report, is independent of the report's realization (see e.g., Ewert and Wagenhofer, 2010; Fischer & Verrecchia, 2000; Fishman and Hagerty 1992). In the standard rational expectations models, this is true because all random variables in the

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still a matter of concern for regulators. The FASB sets, in Statement of Financial Accounting Concepts (SFAC) #2 on the Qualitative Characteristics of Accounting Information, the requirement that accounting information should represent what it purports, namely should be free of bias.

models are either conditional or unconditional normally distributed variables whose variances, as is well known, are independent of realized or reported earnings.

The last result of this section shows that a better ICS does not guarantee a lower residual uncertainty but on the contrary leads to greater uncertainty for sufficiently high reports. Thus a greater earnings quality goes hand in hand with greater uncertainty ex-post for sufficiently high reported earnings.

**Corollary 3** *Improving the firm's ICS (increasing  $\gamma$ ) may increase investors's uncertainty ex-post. Formally, for any two  $\gamma > \gamma'$ , associated with reports  $\tilde{r}_\gamma$  and  $\tilde{r}_{\gamma'}$ , there exists a cutoff  $r^* > c$  such that for all  $r > r^*$ ,  $\text{var}(\tilde{x}|\tilde{r}_\gamma = r) > \text{var}(\tilde{x}|\tilde{r}_{\gamma'} = r)$ .*

Corollary 3 is perhaps also surprising: adopting a better ICS may make investors even more uncertain about the firm's true earnings for sufficiently high reported earnings. This occurs because, as we explain in Section 3.3 below, better ICS firms are more likely to issue higher reports when their ICS is ineffective. Thus, a sufficiently higher report is a stronger signal that the firm's ICS was weak and the report was fake, when the quality of the firm's ICS is higher.

### 3.2 The validity of empirical measures of earnings quality

The empirical measurement of accounting quality has been a major concern to accounting research. Unfortunately, as Ewert & Wagenhofer (2010), put it: "surprisingly, earnings quality is quite vague a concept." There are at least eight measures of earnings quality and all of them remain controversial among empiricists (see e.g. Holthausen and Watts, 2001). Still, with the notable exception of Ewert and Wagenhofer (2010), we have little theory that can map these proxies to primitive notions of information quality.

In this section, I assess the validity of three standard empirical measures of earnings quality in the context of my model: a market based measure, such as the volatility of prices around the announcement, and two accounting based measures: the predictability of earnings and the smoothness of earnings. I contrast these measures against  $\gamma$ .<sup>16</sup> I postpone the discussion of another standard measure of earnings quality commonly

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<sup>16</sup>Neither do I evaluate accrual based measures (see e.g., Jones, 1991, Dechow, Sloan and Sweeney, 1995 and Dechow & Dichev, 2002) nor Basu's (1997) so-called timely loss recognition measure because the model is not rich enough to provide a fair assessment of these measures.



known as the persistence of earnings until I develop a multi-period extension of the model in Section 4,

First, consider the volatility of prices around the earnings announcements. Beaver (1968) was the first to develop the idea that this metric reflected the informational content of earnings. He thought that under the hypothesis that markets are (semi-strong) efficient more informative financial statements should translate into a greater price variability around the earnings announcement. The following result confirms Beaver's (1968) intuition.

**Corollary 4** *The variance of prices  $\text{var}(P)$  around the earnings announcement increases in the firm's earnings quality.*

The result that better earnings quality leads to more volatile stock prices around the earnings announcement is simply an implication of integral precision and holds not only in our setting but generally. In other words, if we adopt the criterion of integral precision as the definition of information quality and adhere to the idea that markets are semi-strong efficient, then a greater volatility of stock prices around the earnings announcement must be considered indicative of better earnings quality.<sup>17</sup>

Second, consider the so-called Smoothness metric of earnings quality. Smoothness has been defined as the relative absence of variability in earnings and usually measured as the ratio of the standard deviation of cash flows over the standard deviation of reported earnings (see Francis, LaFond, Olsson and Schipper, 2004). Regarding this metric, Ewert and Wagenhofer (2010) argue that it is not even clear whether a greater Smoothness should be interpreted as indicative of higher or lower earnings quality. Corollary 5 goes even further: the problem with this metric is not just one of interpretation but that the relation between earnings quality and the smoothness of earnings is ambiguous.

**Corollary 5** *The variance of reported earnings  $\text{var}(\tilde{r})$  is non monotonic in  $\gamma$ .*

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<sup>17</sup>The volatility of prices induced by the earnings announcements as a measure of earnings quality is related to what is deemed the *value relevance* of earnings and often measured either from the coefficient of earnings in a regression of stock returns on earnings or from the R-squared of that same regression (see Francis, LaFond, Olsson and Schipper, 2004). These metrics have become unpopular among empiricists in the last decade (e.g. see Holthausen & Watts, 2001).

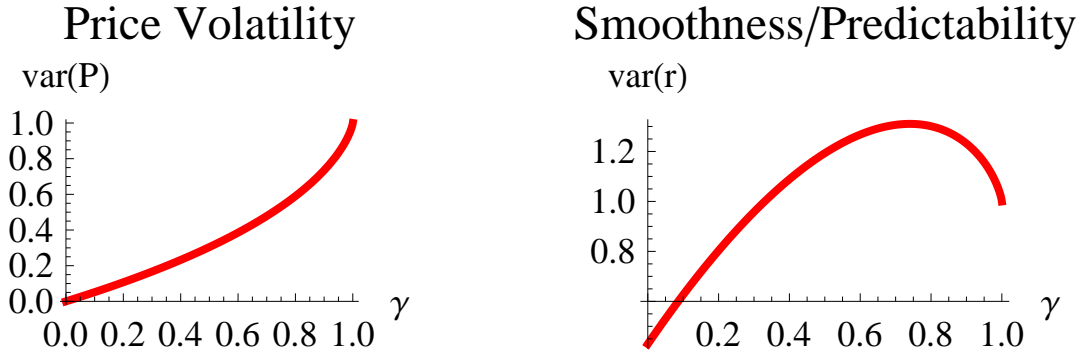


Figure 1: Standard Measures of Earnings Quality.  $\tilde{x} \sim N(0, 1)$ . The volatility of stock prices (price volatility) monotonically increases in  $\gamma$ , going from 0 when  $\gamma = 0$  to  $\text{var}(\tilde{x})$  when  $\gamma = 1$ . By contrast, the variance of the report is non monotonic in  $\gamma$ ;  $\text{var}(\tilde{r})$  is relatively low at low levels of  $\gamma$ , it is maximized at intermediate levels of  $\gamma$  and converges to  $\text{var}(\tilde{x})$  when  $\gamma = 1$ .

This lack of monotonicity implies that the volatility of earnings is not a proper metric of earnings quality: highly volatile earnings are compatible with both relatively high and low earnings quality. The reason for this ambiguity follows. Conditional on a weak ICS, earnings are smoother because the manager concentrates his reports in a relatively small set, i.e., the right tail. This set shrinks as  $\gamma \rightarrow 1$ . At the same time, conditional on a weak ICS, earnings are larger in expected value. These two effects affect the unconditional volatility of reported earnings in opposite directions. Numerical analysis (see Figure 1) reveals that the first effect dominates at low levels of  $\gamma$  inducing relatively smooth earnings, but the second effect dominates at high levels of  $\gamma$  inducing relatively volatile earnings reports.

A second accounting based metric of earnings quality that is closely related to  $\text{var}(\tilde{r})$  is Predictability. Predictability is defined as the ability of prior earnings reports to predict future earnings reports and has been commonly viewed as a desirable attribute of earnings. Ewert and Wagenhofer (2010) operationalize the notion of Predictability by looking at the variance of an earnings report conditional on investors' information set. In a dynamic setting, this measure could be measured by the variance of a report conditional on the history of reports. In the absence of history though –as in the static setting I consider in this section– Predictability should be measured by  $\text{var}(\tilde{r})^{-1}$ . Thus, given the non-monotonic relation between  $\text{var}(\tilde{r})$  and  $\gamma$ , the same reason that invalidates

Smoothness as a metric of earnings quality applies to Predictability.

Conceptually, the justification for this metric as a desirable attribute of reported earnings is not clear. The fact that one can better predict a report does not make the report necessarily more informative about the firm's underlying performance. Ultimately, investors' goal is not to predict the report, but rather to predict what the report is about, namely the firm's true financial performance. In fact, manipulation is always (partially) predictable in equilibrium, shocks by definition are not. In the model, reported earnings that are very predictable may indicate low earnings quality. Relative to the case in which the firm's ICS is perfect, the report may become relatively more predictable when the manager manipulates the information strategically. Thus, contrary to some conventional interpretations, it would appear that predictability is not really a desirable attribute of reported earnings.

### 3.2.1 Estimating earnings quality

In this section I show how  $EQ$  can be estimated using data on earnings announcements. Figure 2, shows one attempt at estimating the cross sectional distribution of  $\gamma$  using COMPUSTAT quarterly data for the last two decades.

I use quarterly earnings announcements as the basic observation. For each firm  $i$  I estimate the probability  $\gamma_i$ . First I construct the normalized earnings  $z_{it} = \frac{r_{it} - \mu_i}{\sigma_i}$ , where  $r_{it}$  is income before extraordinary items in quarter  $t$  for firm  $i$  (COMPUSTAT item IBQ). As proxies for the mean  $\mu_i$  and variance  $\sigma_i^2$  of the true earnings process, I use the sample average  $\hat{\mu}_i$  of net cash flows from operating activities (COMPUSTAT item OANCFQ) and the empirical variance  $\hat{\sigma}_i^2$  of net cash flows from operating activities. As the estimation method, I use the *method of moments* (see, e.g., Hayashi, 2000 Chapter 3). More specifically, I use the first normalized moment  $m_1(\gamma_i) \equiv E(\tilde{z}_i | \gamma_i)$ . Given our knowledge of the density of reported earnings  $g(r | \gamma_i)$  one can obtain  $m_1(\gamma_i) = \int_{-\infty}^{\infty} r g(r | \gamma_i) dr = \frac{\gamma_i \Phi(-k_i)}{k_i}$ . Now, the idea of the method of moments is to match the theoretical moment  $m_1(\gamma_i)$  to its empirical analogue  $\bar{z}_{iT} = \frac{\sum_t z_{it}}{T}$ . Thus using  $m_1(\gamma_i)$  I can obtain  $\hat{\gamma}_i$  by numerically solving for  $\hat{\gamma}_i$  in  $m_1(\hat{\gamma}_i) = \bar{z}_{iT}$ . To obtain  $\hat{\gamma}_i$  I use MATLAB algorithm FMINCON setting  $\frac{1}{2}$  as the starting value (the results virtually did not change as I varied the starting value). By the Delta method (see e.g., Green 2000, page 118), the asymptotic distribution of  $\hat{\gamma}_i$ , is given by  $\sqrt{T}(\hat{\gamma}_i - \gamma_i) \xrightarrow{d} N\left(0, \frac{\sigma^2}{[m_1'(\gamma_i)]^2}\right)$ . A consistent estimator

for the asymptotic variance of  $\hat{\gamma}_i$ ,  $Asy. \text{var}(\hat{\gamma}_i | \mathbf{z}_{iT})$ , is  $Asy. \widehat{\text{var}}(\hat{\gamma}_i | \mathbf{z}_{iT}) = \frac{\sum_{t=1}^T (z_{it} - \bar{z}_{iT})^2}{(T-1)[m_1'(\hat{\gamma}_i)]^2}$  (see e.g., Green, 2000, page 143).

Table 1 presents the results of the estimation attempt. The table contains the mean,

Table 1: The distribution of gamma over time

	mean	sd	skew	kurt	p1	p5	p50	p95	p99
gamma 90-95	0.87	0.23	-2.21	7.53	0.00	0.33	0.99	1.00	1.00
gamma 96-01	0.85	0.24	-2.02	6.61	0.00	0.26	0.97	1.00	1.00
gamma 02-09	0.83	0.26	-1.86	5.61	0.00	0.16	0.96	1.00	1.00

standard deviation (sd), skewness (skew), kurtosis (kurt) and five percentiles of the distribution of  $\gamma$ .

The results suggest that the average  $\hat{\gamma}$  has declined over time going from 0.87 in the early 1990's to 0.83 in the period 2002-2009. Also, the standard deviation of  $\hat{\gamma}$  seems to have increased over time going from 0.23 in the 1990 to 0.26 in 2002-2009. A simple test of comparison of means suggests that these differences are statistically significant. It is left to future empirical research to refine the estimation procedure and perform the necessary robustness checks.<sup>18</sup>

### 3.3 The incidence and size of frauds

The notion of earnings quality is also naturally tied to the occurrence and magnitude of frauds. In this subsection, I refer to the earnings reports issued under a weak ICS as fraudulent reports. The main result demonstrated here is that large frauds are more likely to be associated with institutional environments where firms' ICS are strong whereas small frauds are associated with institutional environments where firms' ICS are weak. As an intermediate step towards this result, the next lemma shows that the value of  $c$  indexes the expected magnitude of the report under a weak ICS,  $E[\tilde{r}|w]$ . In the following lemma, I made explicit the dependence of  $\varphi(\cdot)$  on  $c$  by writing  $\varphi(\cdot; c)$  where  $c$  is defined in Proposition 1.

<sup>18</sup>The estimation can be improved in several dimensions, notably refining the proxies  $\hat{\mu}_i$  and  $\hat{\sigma}_i$ . We could also consider using additional moment conditions so to estimate all three parameters  $\mu, \sigma, \gamma$  from the time series of reported earnings.

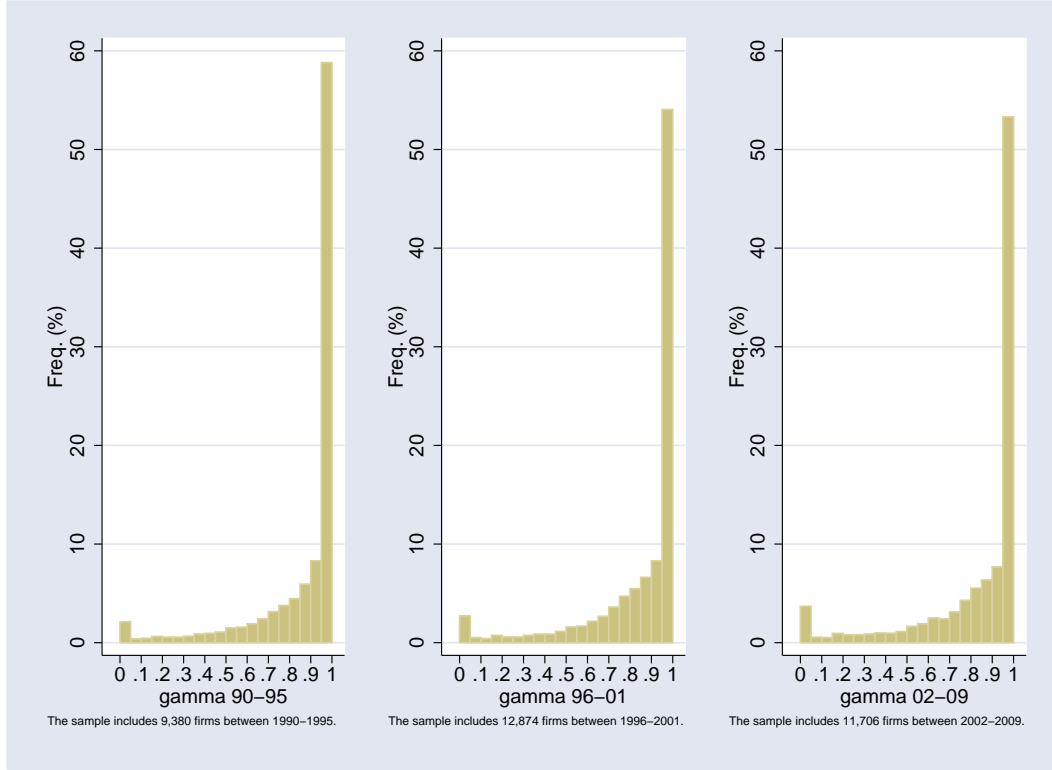


Figure 2: An estimation of the cross sectional distribution of  $\gamma$

**Lemma 1** *The family of equilibrium probability densities  $\{\varphi(\cdot; c)\}_{c \in \mathbb{R}_+}$  satisfies the MLRP property. That is, for any two  $\gamma$  and  $\gamma'$  leading to  $c$  and  $c'$  such that  $c > c'$  the ratio  $\frac{\varphi(r; c)}{\varphi(r; c')}$  is strictly increasing in  $r$ .*

Since MLRP implies first order stochastic dominance (see Shaked and Shantikumar, 2007), Lemma 1 implies that a larger  $c$  leads to a more aggressive reporting under a weak ICS. Armed with this lemma, we can now consider the determinants of the magnitude of frauds.

**Proposition 3** *The expected magnitude of frauds  $EMF \equiv E[\tilde{r} - \tilde{x} | \tilde{\tau} = w]$  is increasing in (i) the quality of the firm's ICS and (ii) in the volatility of earnings  $\sigma$ .*

The intuition for Proposition 3 (i) is simple: for any given report, increasing the quality of the firm's ICS induces investors to reduce the "credibility discount" they apply to the report, as they believe frauds are now less likely. That makes the manager bolder in his reporting strategy: as he enjoys more credibility ex-ante, he can overstate

the firm's earnings more aggressively without fear that investors will disregard the value of the report.

Proposition 3 shows that it is precisely the quality of the firm's ICS that creates the possibility of large scale frauds inasmuch as it creates investors' trust in the firm's accounting report. At a macro level, if we interpreted  $\gamma$  as representing the quality of the institutional environment, as given by the legal and regulatory mechanisms that prevent managers from manipulating the information, Proposition 3 would suggest that the observation of few but large frauds reveals a good institutional environment where firms' ICS is effective with a high probability. In contrast, the observation of many small frauds indicates a weak institutional environment where firms' ICS is effective with low probability.

Proposition 3 (ii) also states that the expected magnitude of frauds is greater as  $\sigma$  gets larger. This is intuitive once we recognize that  $\sigma$  is a measure of the extent of the information asymmetry between the manager and the market. The more uncertain investors are about the firm's earnings  $\tilde{x}$ , the greater the manager's room for report manipulation. A greater  $\sigma$  makes greater values (as well as smaller values) more likely, so that a higher report becomes also more credible as  $\sigma$  increases.

Since  $\sigma$  can represent not just the volatility of earnings, but also the volatility of cash flows *conditional* on investors' information set prior to the release of the manager's report,  $\sigma$  can be thought of as combining two sources of uncertainty: (i) the inherent operational risk of the firm, i.e., the risk that comes from being say in the pharmaceutical or technological industry, and (ii) the relative absence of other independent sources of information about the firm such as analyst following, good credit ratings etc. When the lack of other sources of information results in greater market uncertainty, the manager has more latitude to manipulate the firm's reported earnings, which he will exploit by reporting higher values when the firm's ICS is weak.

Finally, note that even though an improvement in the firm's ICS quality leads to larger frauds on average, reported earnings are on average lower as the firm's ICS improves.

**Definition 4** Denote by  $G_\gamma(r) \equiv \Pr(\tilde{r} \leq r|\gamma)$  the c.d.f. of the firm's earnings reports when the quality of the firm's ICS is  $\gamma$ .

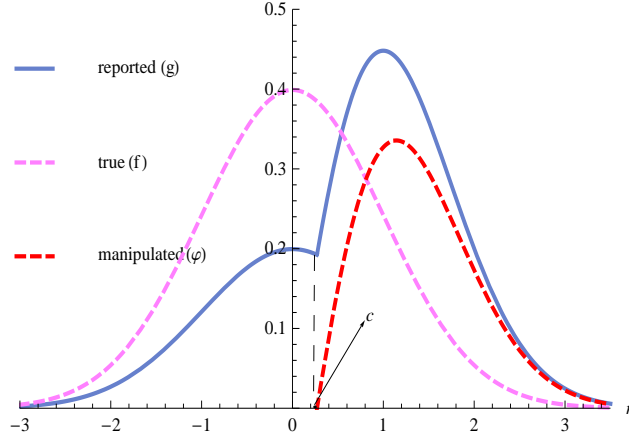


Figure 3: The distribution of reported earnings. The plot assumes  $x_1 \sim N(0, 1)$  and  $\gamma = 0.5$ . Under these assumptions  $c = 0.27 > E(\tilde{x}_1) = 0$ . The value of  $c$  marks the lower limit of the manager’s reporting region under a weak ICS, i.e., the right tail of the distribution of reported earnings.

**Corollary 6** *If  $\gamma \geq \gamma'$ , then  $G_{\gamma'}(\cdot) \succ_{FOSD} G_{\gamma}(\cdot)$ .*

### 3.4 The location of kinks in earnings distributions

Before concluding this section, I note that the reporting game this paper studies also has implications for recent research on the location of so-called “kinks” in earnings distributions. Recall that the value  $c$  is the smallest “managed” earnings the firm reports.  $c$  is also the location of a kink in the distribution of reported earnings, because it is the point at which the true earnings distribution (relevant for the earnings report when the firm’s ICS is strong) and the “fraudulent” earnings distribution (relevant when the ICS is weak) mix. (See Figure 3 below.)

The location of the kink has the following properties:

**Corollary 7** *The equilibrium value of the kink  $c$  is characterized as follows: (i)  $c > \mu$ ; (ii)  $\frac{\partial c}{\partial \gamma} > 0$ ; and (iii)  $\lim_{\gamma \rightarrow 1} c = \infty$  and  $\lim_{\gamma \rightarrow 0} c = \mu$ .*

Corollary 7 is consistent with several empirical facts. First, it shows that  $c$  is always above investors’ prior beliefs about the mean of the distribution of earnings (i.e.,  $c > \mu$ ) confirming the empirical observation that managers tend to “beat market expectations” when they have discretion in reporting their firm’s earnings.

DeGeorge, Patel and Zeckhauser (1997) showed that the empirical distribution of earnings announcements exhibits a “hump” over market’s prior expectations about the earnings report (whose expectations they proxy using analysts earnings forecasts).<sup>19</sup> The authors interpret this finding as evidence of firms engaging in report manipulation taking place in the vicinity of the market’s priors: managers would tend to overstate earnings particularly when the true earnings fall short of what the market expects them to be. In the present model, the distribution of reports also displays a hump starting at  $c$ , as we can see in Figure 3.<sup>20</sup> But note the difference between the results. DeGeorge, Patel and Zeckhauser (1997) find that managers tend to beat the earnings announcement  $E(\tilde{r})$  whereas I find that managers tend to beat the true underlying earnings  $E(\tilde{x})$ .

Beating the market’s expectations does not guarantee that the manager’s report will be able to induce a price increase especially when the firm’s ICS has a low quality ( $\gamma$  is low). To see this, note that  $c$  also defines an upper bound for the firm’s stock price –  $c$  is the maximum price the firm can attain. This maximum price  $c$  is a strictly increasing function of the quality of the firm’s ICS,  $\gamma$ . However, as  $\gamma \rightarrow 0$ , the maximum price  $c$  approaches market’s priors  $\mu$ . In other words, as the firm’s ICS worsens, less information is transmitted from the firm to the market in the earnings report, and eventually no information gets transmitted at all.

## 4 Multiple periods with restatements

In this section, I introduce a multi-period version of the reporting model described in Section 2 as a means to study (1) the manager’s incentives to issue a restatement, (2) how the book-to-market ratio and the income statement interact, (3) how financial

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<sup>19</sup>Guttman et al. (2006) analytically derive a similar result in a model with a quadratic cost of misreporting similar to that in Fischer and Verrecchia (2000). They examine partially separating equilibria in which the manager’s reporting strategy overstates the true earnings when the realization of earnings lies on the tails (either left or right). Pooling occurs in the middle range of the reported earnings. In that range, the manager creates some noise but also saves some misreporting costs. In equilibrium, the distribution of reported earnings displays both a hole in the middle and a mass point.

<sup>20</sup>The distribution of earnings is a mixture whose density  $g(r|\gamma, \mu, \sigma)$  is given by

$$g(r|\gamma, \mu, \sigma) = \begin{cases} \gamma f(r|\mu, \sigma) & \text{if } r < c \\ \gamma f(r|\mu, \sigma) + (1 - \gamma) \varphi(r|\mu, \sigma) & \text{if } r \geq c \end{cases} .$$

All the parameters of the distribution of reported earnings are statistically identified and can thus be estimated using either maximum likelihood or GMM (See Hayashi, Chapter 8).



reporting evolves over time, and (4) the extent to which the persistence of earnings is a valid measure of earnings quality.

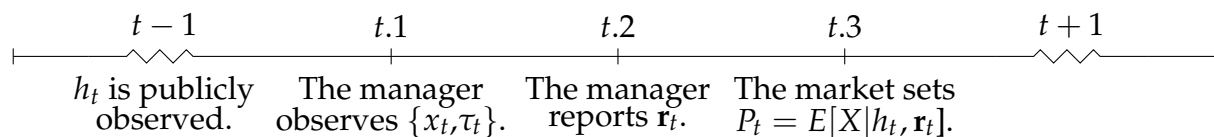
The sequence of events follows. In each period  $t \in \{1, 2, \dots, T\}$ , three events take place. First, the manager privately observes the realization of two i.i.d. and mutually independent variables: the firm's current earnings  $x_t \sim F(x)$  and whether the firm's current ICS  $\tilde{\tau}_t$  is weak ( $\tau_t = w$ ) or strong ( $\tau_t = s$ ). The probability that the firm's ICS is strong (respectively weak) in a given period is  $\gamma$  (respectively  $1 - \gamma$ ).<sup>21</sup>

Second, the manager releases a report  $\mathbf{r}_t$  about the firm's earnings which may include a restatement of any prior report (as described below). We assume that, when the ICS is not effective, the manager selects the report  $\mathbf{r}_t$  so as to maximize the firm's current stock price  $P_t$ .

Third, the market opens and updates the stock price  $P_t$  using all publicly available information, in particular, the firm's entire history of reports  $h_t$ .

To simplify the exposition, I assume that there are no dividends distributions, no discounting, and that the firm's retained earnings constitute the firm's only assets. The structure of this game is common knowledge.

Figure 4: Time-line



There are three natural ways in which restatements can be introduced in the model. All three are symmetric under a strong ICS: when the firm's ICS is strong, truthful restatements are automatically announced if there was a misstatement in the past. The differences among these ways relate to the amount of discretion the manager has over the firm's restatements when the firm's ICS is weak.

**Definition 5** (i) We say that the manager has *full discretion* over restatements if, under a weak ICS, the manager not only has the option to choose whether or not to restate prior reports

<sup>21</sup>The assumption that earnings are i.i.d allows us to emphasize that serially correlated earnings reports can arise even when the underlying earnings process is i.i.d. I relax this assumption in the Appendix.

The assumption that the ICS is i.i.d makes the analysis more tractable We discuss the case of a sticky ICS in the Appendix.

but also has discretion about the content of restatements. (ii) We say that the manager has **no discretion** over restatements if, under a weak ICS, the manager cannot restate prior reports.

The no discretion case describes situations in which restatements are initiated and enforced by an independent third party, like the SEC or an independent auditor. The full discretion case covers the situations in which restatements may sometimes be initiated voluntarily by the manager and are potentially subject to managerial manipulation. In this section I study the full discretion case and relegate to the Appendix the no discretion case.

Consider the full discretion case. When the manager can restate prior reports, the firm's report in any given period,  $\mathbf{r}_t$ , consists of both a statement of the firm's current period earnings and either a confirmation or a restatement of some or all of the firm's prior reports. As noted previously, when the ICS is strong ( $\tau_t = s$ ) the firm must truthfully report both the current earnings and correct any prior misstatement.<sup>22</sup> So if some past reports were fraudulent, the current period report corrects those misstatements. If the ICS is weak, however, the manager not only has the opportunity to misstate the current earnings but also to strategically manipulate the restatements. In particular, when the ICS is not effective a restated report need not be correct.<sup>23</sup> Hence, while restatements may align prior reports with true past earnings realizations when the ICS is effective, restatements may potentially diverge even farther from true past earnings when the ICS is not effective. Investors cannot be certain of whether a restatement was produced under an effective or ineffective ICS, as the ICS's effectiveness is not directly observable.

Clearly, the possibility to restate prior reports makes reporting strategies richer and potentially very complex with a dimensionality that grows period by period. Since the manager can eventually restate any prior report, a report here is a vector with as many

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<sup>22</sup>The Financial Accounting Standards Board's Statement No. 154, Accounting Changes & Error Corrections, applies to financial statements covering fiscal years ending after December 31, 2006. FASB Statement 154 requires companies to restate a previously issued financial statement when the correction of an error is necessary. The old rules (in APB Opinion No. 20) required companies to report the cumulative effect of the change as part of current-year income but not to restate the prior year's report. Readers could compare consistent data only by reviewing the pro forma information in the footnotes.

<sup>23</sup>The restatements database known as *Audit Analytics* which covers restatements of public US firms since January 2001 shows a total of 31,452 restatements, many of them including several items. This number corresponds to 3.03% of the number of quarter-firm observations found in COMPUSTAT during the period 2001-2009. 6.8% of those restatements are restatements of restatements, namely restatements of financial statements that have been previously restated.

entries as periods have elapsed since the beginning of the firm's operation. However, we can assume without loss of generality that a report  $\mathbf{r}_t$  consists only of two elements, denoted by  $B_{t-1}^t$  and  $r_t^t$ , where  $B_{t-1}^t$  is a confirmation/restatement of the prior book value and  $r_t^t$  is a statement about the firm's current earnings. I also denote by  $B_t \equiv B_{t-1}^t + r_t^t$  the firm's book value in period  $t$  after the release of the manager's report. Consistent with this notation a restatement occurs whenever  $B_t^{t-1} \neq B_{t-1}$ .

This two dimensional representation of the manager's report can be further simplified. The following lemma shows that the manager's reporting problem is, in essence, single dimensional. Lemma 2 establishes that restating information cannot be informative in equilibrium and that  $\mathbf{r}_t = \{B_{t-1}^t, r_t^t\}$  contains the same information as the total book value of retained earnings,  $B_t$ . We denote by  $X_T = \sum_{j=1}^T x_j$  the firm's value, i.e., the sum of the firm's earnings.

**Lemma 2** *In equilibrium, for every period  $t$  and history  $h_t$ ,  $E[X_T | \mathbf{r}_t, h_t] = E[X_T | B_t, h_t]$  for all  $\mathbf{r}_t$ .*

Lemma 2 states that conditional on the history  $h_t$ , investors' expectations about  $X$  can only depend on the total book value  $B_t$  in equilibrium. In other words, how  $\mathbf{r}_t$  is broken into its individual components  $\{B_{t-1}^t, r_t^t\}$  is irrelevant for the purpose of estimating  $X$ . An implication of Lemma 2 is that, given both the history  $h_t$  and the book value  $B_t$ , whether there is a restatement of past reports or not is by itself uninformative. Alternatively put, a restatement of a past report is, by itself, not informative about the current state of the firm's ICS, given both the current book value  $B_t$  and the history of past reports. More generally, Lemma 2 implies that, relative to the information contained in  $\{h_t, B_t\}$ , a restatement cannot provide any additional information about  $X$  and therefore cannot have price consequences. Note that the sufficiency of  $B_t$  in the estimation of  $X$  can only hold if conditional on  $B_t$  the probability of a restatement is the same under both a weak and a strong ICS. Conditional on the book value the manager's propensity to restate information is independent of whether or not the ICS is effective in the current period.

Lemma 2 entails an important simplification which allows us to solve the model as if in each period  $t$  the manager reports only the total book value  $B_t$  instead of the vector  $\{B_{t-1}^t, r_t^t\}$  or, alternatively, as if the manager only reports an increment over the firm's

previous book value  $B_{t-1}$ . In fact, we can think of the manager's reporting strategy as a two-step decision problem whereby in the first step the manager chooses the book value increments that will be implicit in his report and in the second step the manager selects the specific entries of  $\{B_{t-1}^t, r_t^t\}$ . By Lemma 2, the second step is uninformative and generates no price consequences, so I will focus on the first step.

I will state the equilibrium in terms of  $\varphi_t(\cdot|h_t)$  defined as the probability density of the incremental book value that the manager (implicitly) reports in the actual report  $\mathbf{r}_t$ .<sup>24</sup> I will denote this reported increment by  $r_t \equiv B_t - B_{t-1}$ , but note that  $r_t$  represents the firm's current earnings net of prior misstatements. The p.d.f  $\varphi_t(\cdot|h_t)$  will be referred to as the equilibrium reporting strategy.

I adopt PBN as the equilibrium concept. For simplicity, throughout this section, I normalize  $\tilde{x}_t \sim N(0, \sigma)$ .<sup>25</sup> (Note that this normalization implies that the stock price  $P_t$  will always be lower than the book value  $B_t$  given that the true earnings are i.i.d.)

**Proposition 4** *In each period  $t$ , given  $h_t = \{r_1, r_2, \dots, r_{t-1}\}$ , there is a unique PBN equilibrium characterized as follows: i) if  $\tau_t = s$  the manager truthfully reports an increment  $r_t = X_t - B_{t-1}$ ; and ii) if  $\tau_t = w$ , the manager's reporting strategy  $\varphi_t(\cdot|h_t)$  is given by the following p.d.f:*

$$\varphi_t(r_t|h_t) = \begin{cases} 0 & \text{if } r_t < c_t \\ \frac{\gamma}{1-\gamma} \frac{r_t - c_t}{c_t + B_{t-1} - p_{t-1}} f_t(r_t|h_t) & \text{if } r_t \geq c_t \end{cases} \quad (10)$$

where  $c_t = c(h_t)$  is implicitly defined by the following equation

$$\int_{c_t}^{\infty} \varphi_t(r_t|h_t) dr_t = 1. \quad (11)$$

and  $f_t(r_t|h_t) \equiv \frac{\partial \Pr(\tilde{r}_t \leq r_t | \tau_t = s, h_t)}{\partial r_t}$ . The pricing function is given by

$$P_t = B_{t-1} + \min(r_t, c_t) \quad (12)$$

It is apparent from Proposition 4, and in particular from consideration of equation (10), that the possibility of restating prior reports introduces serial correlation in the time series of reports, linking the reporting strategy  $\varphi_t(\cdot)$  to the firm's history of reports

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<sup>24</sup>Of course, this is not a restriction on the manager's reporting strategy: once we obtain the equilibrium distribution of the increment we can specify the distribution of the full report  $\mathbf{r}_t$  using Lemma 2.

<sup>25</sup>This normalization is only needed to simplify the expressions. Without this normalization, an additional state variable, tracking the expected value of future earnings, would be required.

$h_t$ . More specifically, we see that the history  $h_t$  affects the current report through the gap between book and market values  $B_{t-1} - p_{t-1}$ . Note that whenever investors are certain that the firm's ICS was effective in the prior period, i.e., when  $\gamma_{t-1} = 1$ , then the price and the book value coincide  $B_{t-1} = p_{t-1}$ . When this is the case, the reporting strategy  $\varphi(\cdot|h_t)$  becomes independent of the past, being identical to the reporting strategy of the static model, described by Proposition 1.

The pricing function described by equation (12) follows the same principles verified in the single-period model, being flat over the right tail of the distribution of reports,  $[c_t, \infty)$ . Initially, it may appear that the pricing function takes the firm's book  $B_{t-1}$  at face value (note that the intercept of the pricing function is  $B_{t-1}$  both in the left and in the right tail). Yet careful consideration of equation (12) reveals that only a portion of  $B_{t-1}$  is actually impounded in the current stock price (recall that  $c_t$  is a function of  $B_{t-1}$ .) This becomes apparent once we express the pricing function as:

$$P_t = \gamma_t B_t + (1 - \gamma_t) p_{t-1} \quad (13)$$

where  $\gamma_t = \Pr(\tau_t = s | \mathbf{r}_t, h_t)$ . This representation of the pricing function shows that both  $B_t$  and  $B_{t-1}$  are impounded in the price only to the extent the current book is credible, namely with probability  $\gamma_t$ . If the current report had no credibility at all ( $\gamma_t = 0$ ) the current price  $P_t$  would only reflect historical information, as captured by the prior price  $p_{t-1}$ .

## 4.1 The persistence of earnings

The multi-period model provides an opportunity to examine the validity of another empirical measure of earnings quality: the so-called Persistence metric. Persistence, has been defined as the autocorrelation of earnings reports, which I denote here by  $\rho \equiv \frac{\text{cov}(r_t, r_{t-1})}{\sqrt{\text{var}(r_t)} \sqrt{\text{var}(r_{t-1})}}$  (see Francis, LaFond, Olsson and Schipper, 2004). In the present model, the stochastic process of earnings is i.i.d., hence reported earnings would not be persistent if the firm's ICS was perfect ( $\gamma = 1$ ).<sup>26</sup> To examine the validity of the Persistence metric, I consider how imperfections in the firm's ICS can affect the persistence of reported earnings. In particular, I study whether these imperfections induce positive or negative autocorrelation.

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<sup>26</sup>This assumption is a normalization; what is relevant is how the firm's ICS affects the persistence of reported earnings conditional on a given persistence of the underlying earnings.

As previously noted, whenever book and market values coincide ( $B_{t-1} - p_{t-1} = 0$ ) the firm's reporting becomes independent of the history. However, as soon as there is a gap between book and market values ( $B_{t-1} - p_{t-1} > 0$ ), the time series of reported earnings display serial correlation.

The next corollary establishes that a large gap between book and market values induces a less aggressive reporting behavior in the present.<sup>27</sup>

**Corollary 8** *Reporting becomes less aggressive as the gap between book and market values  $B_{t-1} - p_{t-1}$  increases. Formally,  $\frac{\partial E(\tilde{r}_t|h_t)}{\partial B_{t-1}} < 0$ .*

A gap between book and market values reflects in the model that the firm's book value is not entirely credible.<sup>28</sup> A history of large reports that led to a wide gap between book and market values (i.e., to a large book-to-market ratio) results in a less aggressive reporting behavior in the present. This result has two components. First, a large book-to-market ratio raises the manager's incentives to "invest" in credibility so as to reduce the credibility discount that affects the book value. By reporting lower values today, the manager is able to increase the stock price by raising the credibility of the book value. Second, a large book-to-market ratio is associated in the model with a high probability of manipulation in the past and is therefore a good predictor of a downward restatement, conditional on a strong ICS. The need to mimic a strong ICS forces the manager, under a weak ICS, to also report lower values.

Corollary 8 supports the idea that when the firm's ICS is not perfect (i.e. when  $\gamma < 1$ ) reporting becomes negatively autocorrelated which in turn suggests that a low persistence of earnings may indicate poor earnings quality. Indeed, numerical analysis confirms that the autocorrelation of earnings reports decreases as  $\gamma$  goes down (see Figure 5).

Note that the negative correlation of earnings announcements is not driven here by an exogenous accrual reversal mechanism (such as that described by Evans and Sridhar,

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<sup>27</sup>Throughout this section, I use the term aggressive reporting in the following sense.

**Definition 6** *In any period  $t$ , given the history  $h_t$ , a reporting system  $\tilde{r}_t$  is said to be more aggressive than another reporting system  $\tilde{r}'_t$  if  $E[\tilde{r}_t|h_t] \geq E[\tilde{r}'_t|h_t]$ .*

<sup>28</sup>Notice that since we assume that accounting rules are not biased and we also assume  $E(x_t) = 0$ , the book to market ratio under a perfect ICS ( $\gamma = 1$ ) is always 1.

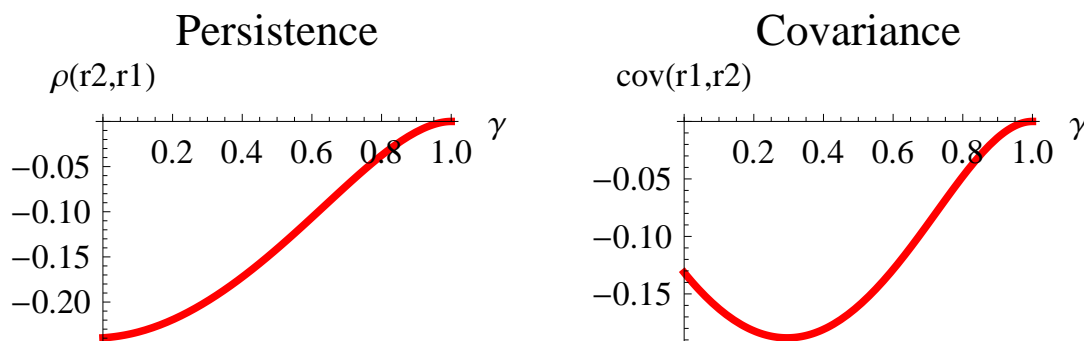


Figure 5: The persistence of reported earnings.  $x_t \sim N(0, 1)$ . Persistence increases in  $\gamma$ , being equal to  $\rho(x_2, x_1)$  when  $\gamma = 1$ .

1996 or by Ewert and Wagenhofer, 2010) but, instead, by the discipline that investors' disbelief imposes on the manager's reporting behavior. Managers whose firms had a history of large reports derive lower marginal benefits from an aggressive reporting behavior in the present, simply because they face a stronger credibility discount than the average firm.

## 5 Concluding remarks

This paper proposes a new measure of earnings quality based on the probability that the firm's internal control system is effective that has strong theoretical foundations and can be estimated using either the time series of earnings or the relation between earnings and prices.

I describe the equilibrium distribution of reported earnings as a mixture that depends on three parameters: two parameters that characterize the distribution of the un-managed earnings process and an additional parameter associated with the quality of the firm's internal control system. All three parameters are statistically identified, so that future empirical research could disentangle the distribution of the un-managed earnings from the distribution of reported earnings.

In a dynamic setup where managers are allowed to restate prior announcements, a natural feedback between the firm's balance sheet and its income statement arises. Among other things, this feedback reduces the persistence of reported earnings, specially when the firm's ICS has low quality. Using this setup, I also evaluate four widely-

used empirical metrics of earnings quality: the volatility of prices around the earnings announcement and the persistence, predictability and smoothness of earnings. I show that neither the predictability nor the smoothness of reported earnings are valid metrics of earnings quality.

## 6 Extensions

### 6.1 When the manager has no discretion over the firm's restatements

In this section I show how the results change when one assumes that restatements are non-manipulable, i.e., when restatements take place only under a strong ICS.

Assume that in any period  $t$ , when the ICS is strong ( $\tau_t = s$ ) the manager truthfully reports the earnings realized up to the current period  $t$  (i.e., the manager reports  $\mathbf{r}_t = \mathbf{x}_t$ ) and when the ICS is weak ( $\tau_t = w$ ) the manager only has discretion to manipulate current income but cannot restate prior reports. Formally, I represent this by assuming the manager is constrained to choose a report from the set  $\{\{\mathbf{r}_{t-1}^t, r^t\} : \mathbf{r}_{t-1}^t = \mathbf{r}_{t-1} \text{ and } r^t \in \mathbb{R}\}$ . Assume that all the other assumptions described in Section 4 hold. Then the next result follows: in every period  $t$ , given  $\mathbf{x}_t$  and  $h_t$ , there is a unique equilibrium in which (i) the manager reports  $\mathbf{r}_t = \mathbf{x}_t$  when  $\tau_t = s$ , and (ii) when  $\tau_t = w$ , the manager confirms the prior report with probability one, i.e.,  $\mathbf{r}_{t-1}^t = \mathbf{r}_{t-1}$ , and reports the current earnings  $r_t^t$  using the probability density

$$\varphi_t^*(r_t^t | h_t) = \frac{\gamma}{1-\gamma} \gamma_{t-1} \frac{r_t^t - c_t^*}{c_t^* + B_{t-1} - p_{t-1}} f(r_t^t). \quad (14)$$

The support of  $\varphi_t^*(\cdot | h_t)$  is  $[c_t^*, \infty)$ , where  $c_t^* = c_t^*(\gamma)$  is defined as the solution to  $\phi(c_t^*) - c_t^* \Phi(-c_t^*) = \frac{c_t^* + B_{t-1} - p_{t-1}}{\frac{\gamma}{1-\gamma} \gamma_{t-1}}$ .

The reporting strategy  $\varphi_t^*$  is similar to that arising when the manager has full discretion over the restatements (see equation 10). Note, however, the extra term  $\gamma_{t-1}$  that multiplies  $\frac{\gamma}{1-\gamma}$  in equation (14). When the prior report  $\mathbf{r}_{t-1}$  has a low credibility level  $\gamma_{t-1}$ , the manager, in the current period  $t$ , inherits that low level of credibility because he is constrained to confirm the prior report. This effect reinforces the manager's incentive to report lower values for the firm's current income  $r_t^t$  in period  $t$ .



## 6.2 When earnings are not i.i.d.

Assume that the true earnings are not i.i.d. but characterized by a general continuous joint p.d.f.  $f(\mathbf{x})$ . Assume that the other assumptions described in Section 4 hold. In each period  $t$  given  $h_t$ , there is an equilibrium characterized as follows: i) if  $\tau_t = s$  the manager truthfully reports an increment  $r_t = X_t - B_{t-1}$ ; and (ii) if  $\tau_t = w$ , the manager's reporting strategy  $\varphi_t^-(\cdot|h_t)$  is given by the following p.d.f.:

$$\varphi_t^-(r_t|h_t) = \begin{cases} 0 & \text{if } r_t < c_t^- \\ \frac{\gamma}{1-\gamma} \frac{r_t - c_t^-}{B_{t-1} - p_{t-1} + c_t^-} f_t(r_t|h_t) & \text{if } r_t \geq c_t^- \end{cases}, \quad (15)$$

where  $c_t^- = c^-(h_t)$  is implicitly defined by the following equation

$$\int_{c_t^-}^{\infty} \frac{\gamma}{1-\gamma} \frac{r_t - c_t^-}{B_{t-1} - p_{t-1} + c_t^-} f_t(r_t|h_t) dr_t = 1, \quad (16)$$

and  $f_t(r_t|h_t) \equiv \frac{\partial \Pr(\tilde{r}_t \leq r_t | h_t, \tau_t = s)}{\partial r_t}$ . The pricing function is given by

$$P_t = B_{t-1} + \min(c_t^-, r_t). \quad (17)$$

Note that  $f_t(r_t|h_t)$  is a function of the whole history  $h_t$  even if  $\gamma_{t-1} = 1$ . Clearly, if  $\gamma_{t-1} = 1$ , the problem simplifies as  $f_t(r_t|h_t)$  becomes  $f_t(r_t|\mathbf{x}_{t-1} = \mathbf{r}_{t-1})$ . If  $\gamma_{t-1} < 1$  and  $\gamma_{t-2} = 1$  then  $f_t(r_t|h_t) = \gamma_{t-1} f_t(r_t|\mathbf{x}_{t-1} = \mathbf{r}_{t-1}) + (1 - \gamma_{t-1}) f_t(r_t|\mathbf{x}_{t-2} = \mathbf{r}_{t-2})$ . But, in general,

$$f_t(r_t|h_t) = \gamma_{t-1} f_t(r_t|\mathbf{x}_{t-1} = \mathbf{r}_{t-1}) + (1 - \gamma_{t-1}) \gamma_{t-2} f_t(r_t|\mathbf{x}_{t-2} = \mathbf{r}_{t-2}) \\ + (1 - \gamma_{t-2}) (\gamma_{t-3} f_t(r_t|\mathbf{x}_{t-3} = \mathbf{r}_{t-3}) + (1 - \gamma_{t-3}) (\gamma_{t-4} f_t(r_t|\mathbf{x}_{t-4} = \mathbf{r}_{t-4}) + \dots)).$$

The general lesson from this case is that the reporting strategy in (17) tends to emulate the statistical properties of the true earnings process conditional on investors' information set at any given period  $t$ . But as before, the reporting strategy is distorted by a factor  $\frac{\gamma}{1-\gamma} \frac{r_t - c_t}{c_t + B_{t-1} - p_{t-1}}$  that depends both on  $\frac{\gamma}{1-\gamma}$  and the gap between book and market values ( $B_{t-1} - p_{t-1}$ ). As in Proposition 4, this distorting factor follows from the manager's objective to maximize the stock price and induces, on average, over-reporting.

## 6.3 When the ICS is "sticky"

In this section I consider the case in which the firm's ICS is perfectly correlated across time periods. Clearly, in this context there is no room for restatements because by restating information the manager would reveal the firm's type which would induce a

sharp decrease in the firm's stock price. Thus, the manager's report consists here only of the firm's current earnings. Note that assuming that the ICS is perfectly correlated is equivalent to assuming that there are two types of firms: a strong ICS firm that is always truthful and a weak firm that always manipulates its reports. Proposition 5 provides the equilibrium assuming  $\text{corr}(\tau_t, \tau_{t-1}) = 1$ .

**Proposition 5** *There is a PBN characterized as follows. For each period  $t$  and each  $h_t$ , (i) the manager's reporting strategy is given by:*

$$\sigma_t(r_t|h_t) = \begin{cases} 0 & \text{if } r_t < c_t \\ \frac{\gamma_{t-1}}{1-\gamma_{t-1}} \frac{r_t - c_t}{c_t + B_{t-1}} f(r_t) & \text{if } r_t \geq c_t \end{cases} \quad (18)$$

where  $c_t = c_t(h_t)$  is the unique solution to

$$\int_{c_t}^{\infty} \sigma_t(r_t|h_t) dr_t = 1 \quad (19)$$

and  $\gamma_0 = \gamma, B_0 = 0$ . (ii) The equilibrium pricing function is

$$P_t = B_{t-1} + \min(r_t, c_t) \quad (20)$$

Proposition 5 (i) provides the equilibrium reporting policy. The manager uses a random reporting strategy  $\sigma_t(\cdot)$  whose support is given by the interval  $[c_t, \infty)$  where the lower limit of that interval  $c_t$  is uniquely determined by equation (19). Proposition 5 (ii) shows that the stock price fully impounds the firm's report (both current and prior) if in any period the firm released a report outside the period's right tail because that behavior fully reveals the firm's ICS is strong. By contrast, if the firm's report falls in the right tail then the stock price impounds the firm's book value  $B_t$  partially, i.e., only to the extent of the firm's credibility.

Consider the dynamics of this model. First, I characterize how investors' beliefs about the firm's type evolve over time as they observe the firm's time series of reports, and how this learning process impacts the behavior of stock prices. Second, I characterize how the firm's reporting strategy evolves over time as both the firm's level of credibility  $\gamma_t$  and the firm's book  $B_t$  change.

Figure 6 previews the results of this section. It depicts the dynamics of the game simulated for  $T = 30$  and  $\gamma = 0.9$  when the firm's ICS is weak. The upper left panel shows the dynamics of the stock price  $P_t$ . There we see a rapid increase in the stock price

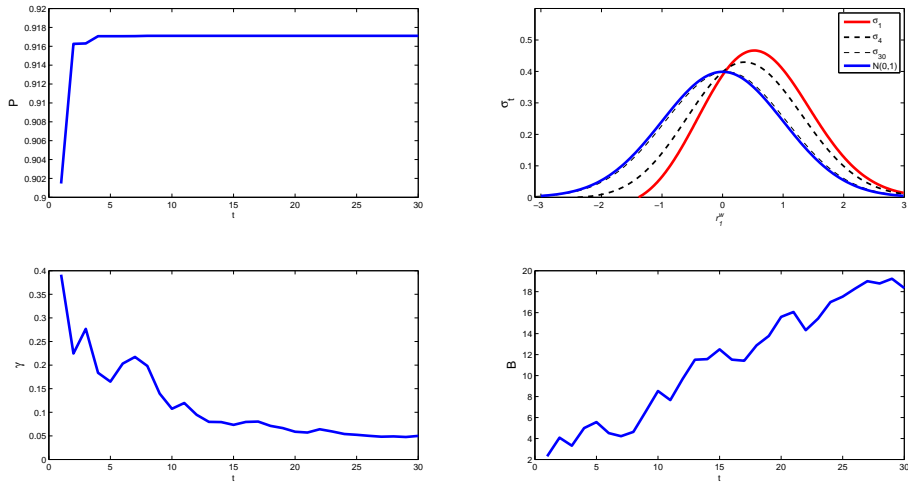


Figure 6: The dynamics of financial reporting. The simulations assume  $\tau = w$  and  $\gamma = 0.9$ . The top right panel simulates  $\sigma_t$  for  $t = 1, 4$  and  $30$ .

in the early stage of the firm’s life up to year 20, followed by almost a completely flat stock price. Note that the price stabilizes around 1.2, namely 1.2 standard deviations above the firm’s true expected value. The bottom left panel, by contrast, depicts the evolution of the firm’s credibility  $\gamma_t$  as randomly converging to zero. These seemingly contradictory results are explained by the bottom right panel, which describes the exploding pattern followed by the firm’s book value  $B_t$ . The weak firm’s book value tends to explode at a faster speed than the speed at which the firm’s credibility shrinks. This is what allows the weak firm to systematically increase the stock price. Finally the upper right panel shows the evolution of the weak firm’s disclosure policy  $\sigma_t$  as converging to  $N(0, 1)$ . Thus, although always uninformative, the reporting policy of the weak firm becomes, on average, almost indistinguishable from that of the strong firm in the long run.

The first result is intuitive. It shows that despite the fact that information is never verifiable, investors get to learn the weak firm’s type. They do it by contrasting the properties of the true stochastic process of earnings, which they know ex-ante, against the observed time series properties of the firm’s disclosures. The weak firm itself progressively reveals its type by the pattern of its reports.

**Proposition 6** *The weak firm's credibility  $\gamma_t = \Pr(\tau = s | h_t |_{\tau=w})$  almost surely converges to zero as  $t$  grows large.*

Investors learn the firm's type by studying the time series of its reports. To identify the firm's type, investors consider the serial correlation as well as the volatility of the firm's disclosures, among other statistics. Note that the pattern of the firm's reporting is as relevant as its magnitude; in other words the firm's size  $B_t$  is not a sufficient statistic for  $\{h_t, B_t\}$  in the estimation of  $\tilde{\tau}$ .

This paper established that investors should come arbitrarily close to learn the firm's type in the long run, however the speed of investors' learning about  $\tilde{\tau}$  is endogenous and partially controlled by the manager. For instance, in the long run the manager of the weak firm mimics very closely the behavior of a strong firm, slowing down investors's learning.

Still, the firm's credibility tends to vanish over time. Yet, the weak firm is able to manipulate the stock price indefinitely. Proposition 7 shows, in fact, that the stock price of a weak firm systematically increases.

**Proposition 7** *A firm's stock price behavior can be characterized as follows: (i) If the firm's ICS is weak, the stock price systematically increases, i.e.,  $P_t |_{\tau=w} \geq P_{t-1} |_{\tau=w} \geq P_1 |_{\tau=w} = c_1 > E[X_T]$  for all  $t \geq 1$ . (ii) For a given report  $r_t$ , the stock price decreases in the firm's book  $B_{t-1}$ , i.e.,  $\frac{\partial P_t}{\partial B_{t-1}} \leq 0$  when  $\gamma_t < 1$ . (iii) The stock price of larger firms (with larger  $B_{t-1}$ ) is less responsive to the firm's report  $r_t$ , i.e.,  $\frac{\partial}{\partial B_{t-1}} \frac{\partial P_t}{\partial r_t} \leq 0$ .*

Proposition 7 (i) establishes that a weak firm is able to systematically increase the stock price by manipulating the earnings report. This pattern, at first blush, might seem at odds with investors' rationality (given Proposition 6), however it is, in fact, an implication of the requirement that the stock price must be a martingale in equilibrium.

Naturally, the manager's capacity to increase the price should vanish as the firm progressively loses its credibility, which explains why the price tends to stabilize at some level, consistent with what we see in the right panel of Figure 6. Of course, the exact level at which the weak firm's stock price stabilizes is in itself random, but its lower bound is known: the unconditional expectation of the firm's total earnings.

As I noted, Proposition 7 (i) seems to contradict Proposition 6. How can investors price a weak firm above its unconditional value and at the same time be almost sure

the firm is weak (which implies the firm's reports are uninformative)? Proposition 8 reconciles these two results. Although the credibility of a weak firm tends to vanish in the long run, its book value  $B_t$  tends to explode at a faster pace so that the stock price ( $P_t = \gamma_t \cdot B_t$ ) increases period by period. In a sense, the weak firm is always ahead of investors' expectations.

**Proposition 8** *The weak firm's reporting policy can be characterized as follows: (i)  $\text{plim} B_t = \infty$  and (ii)  $\frac{\partial E(r_t | h_t, \tau=w)}{\partial B_{t-1}} \leq 0$ .*

One implication of Proposition 8 when combined with Proposition 7 (i) is that the weak firm's market-to-book ratio should decrease over time and eventually become very small.

Note that the disproportionate growth of the firm's book value is compatible with the reporting policy of weak firms evolving towards a relative neutrality in the long run. In the long run, as the firm's book value becomes large, the reporting policy of weak firms will mirror the properties of the true earnings, in the sense that  $r_t |_{\tau=w} \rightarrow N(0, 1)$ . This is explained by the evolution of the manager's incentives: as the firm's book value explodes and the firm's credibility shrinks, the relative importance of persuading the market that the firm's current performance is good becomes negligible compared with restoring the firm's credibility.

Expressing the stock price as

$$P_t = \gamma_t B_{t-1} + \gamma_t r_t, \quad (21)$$

renders these two sources of incentives apparent. The first term reflects the importance of inducing credibility in market beliefs. If  $B_{t-1}$  is large, then the firm experiences strong incentives to be relatively less aggressive as a means of inducing a higher credibility  $\gamma_t$  for the firm's book value  $B_{t-1}$ . The second term reflects the importance of reporting high current earnings. The higher  $\gamma_t$  is and the less sensitive  $\gamma_t$  is to the current report  $r_t$  the larger the weak firm's incentives to issue an aggressive earnings report.

## A Appendix

Throughout the appendix I denote by  $\Phi(\cdot)$  and  $\phi(\cdot)$  the c.d.f. and p.d.f. of the standard normal distribution. I also use  $\theta \equiv \frac{\gamma}{1-\gamma}$ .

**Proof of Proposition 1.** I establish the existence and uniqueness of  $c$  in general, i.e., only assuming that  $f(\cdot)$  is a continuous density with full support over  $[\underline{x}, \bar{x}]$  and that  $E(|\tilde{x}|)$  exists. With a slight abuse in notation, I allow for  $\underline{x} = -\infty$  and  $\bar{x} = \infty$  and define functions and closedness over the extended real line when  $\underline{x}$  or  $\bar{x}$  are infinite. Defining the auxiliary variable  $z = \frac{r-\mu}{\sigma}$ , one can change variables in equation (7) to obtain

$$\frac{\gamma\sigma}{1-\gamma} \int_k^{\frac{\bar{x}-\mu}{\sigma}} \left(\frac{z}{k} - 1\right) f(\mu + \sigma z) dz = 1. \quad (22)$$

where  $k = \frac{c-\mu}{\sigma}$ . The left hand side of equation (22),

$$RHS(k) \equiv \frac{\gamma\sigma}{1-\gamma} \int_k^{\frac{\bar{x}-\mu}{\sigma}} \left(\frac{z}{k} - 1\right) f(\mu + \sigma z) dz,$$

is a strictly decreasing function of  $k$  and is continuous by hypothesis because  $f(\cdot)$  is continuous. On the other hand, it is easy to verify that  $\lim_{k \rightarrow 0} RHS(k) = \infty$  and  $\lim_{k \rightarrow \frac{\bar{x}-\mu}{\sigma}} RHS(k) = 0$ , given that  $E(\tilde{z})$  also exists. Hence, the result follows from the Intermediate Value Theorem. ■

Note that when  $\tilde{x} \sim N(\mu, \sigma)$ , equation (22) boils down to

$$\theta \frac{\phi(k) - k\Phi(-k)}{k} = 1 \quad (23)$$

**Proof of Corollary 1.** We will show that integral precision can be indexed by  $\gamma$ . In particular, an increase in  $\gamma$  means greater integral precision. Denote by  $\tilde{r}_\gamma$  the equilibrium reporting system when the firm's ICS is  $\gamma$ . I will show that the family of signals  $\{\tilde{r}_\gamma\}_{\gamma \in [0,1]}$  is ordered according to integral precision criterion. Take  $\gamma > \gamma'$ . The definition of integral precision says that  $\tilde{r}_\gamma$  is more integral precise than  $\tilde{r}_{\gamma'}$  if  $E[\tilde{x}|r_\gamma]$  is greater in the convex order than  $E[\tilde{x}|r_{\gamma'}]$ , namely for any convex real valued function  $\psi$ ,  $E[\psi(E[\tilde{x}|r_\gamma])] \geq E[\psi(E[\tilde{x}|r_{\gamma'}])]$ . We will make use of Lemma 1 (i) in Ganuza and Penalva (2010). To do so, we need to apply the probability integral transformation. So I define  $\Pi_j = G_j(\tilde{r}_j)$  for  $j = \gamma, \gamma'$  where  $G_j(\cdot)$  is the cumulative distribution of  $\tilde{r}_j$ . Recall that for a given  $\gamma$ , leading to a threshold  $c_\gamma$ , the c.d.f of  $\tilde{r}_\gamma$  is given by

$$G_\gamma(r) = \begin{cases} \gamma F(r) & \text{if } r \leq c_\gamma \\ \gamma F(r) + \frac{\gamma}{c_\gamma} \int_{c_\gamma}^r t f(t) dt & \text{if } r \geq c_\gamma \end{cases} \quad (24)$$

This transformed signal  $\Pi_j$  is uniformly distributed on  $[0, 1]$ . As both signals  $\Pi_\gamma$  and  $\Pi_{\gamma'}$  have the same marginal distribution, their realizations are directly comparable, regardless of the original distributions of  $\tilde{r}_\gamma$  and  $\tilde{r}_{\gamma'}$ . We define the conditional expectations using the transformed signals by  $W_j(\pi) = E[\tilde{x}|\Pi_j = \pi]$ . Given the pricing function it is easy to calculate

$$W_j(\pi) = \begin{cases} F^{-1}\left(\frac{\pi}{\gamma_j}\right) & \text{if } \pi \leq \gamma_j F(c_j) \\ c_j & \text{if } \pi \geq \gamma_j F(c_j) \end{cases} \quad (25)$$

. We need to show that  $\int_{-\infty}^{\pi} [W_\gamma(p) - W_{\gamma'}(p)] dp \leq 0$  for all  $\pi \in (0, 1)$ . Given that  $\int_0^1 [W_\theta(p) - W_{\theta'}(p)] dp = 0$  (by the law of iterated expectations) then showing that (i) the two functions  $W_\gamma(p)$  and  $W_{\gamma'}(p)$  single cross at some point  $\bar{p} \in (0, 1)$  and that (ii)  $W_\gamma(p) \leq W_{\gamma'}(p)$  for all  $p \leq \bar{p}$  would be sufficient. We know that  $W_j(\pi)$  is an increasing function of  $\pi$ . It is also easy to verify that  $\frac{\partial}{\partial \gamma} \frac{\partial \Phi^{-1}\left(\frac{\pi}{\gamma}\right)}{\partial \pi} < 0$  guarantees (i) and (ii). The crossing point  $\bar{p}$  is implicitly defined by  $F^{-1}\left(\frac{\bar{p}}{\gamma}\right) = c_{\gamma'}$ . ■

**Proof of Corollary 2.** By Bayes rule  $\gamma(r) = \begin{cases} \frac{\gamma f(r)}{\gamma f(r) + (1-\gamma)\varphi(r)} & \text{if } r \geq c \\ 1 & \text{if } r < c \end{cases}$ . The result follows from substituting (11). ■

**Proof of Corollary 7.** Part (i) is implied by the fact that  $k > 0$  (see Proof of Lemma 1). Part (ii) would be established upon showing that  $\frac{\partial k}{\partial \theta} > 0$ . By the Implicit Function Theorem,  $\frac{\partial k}{\partial \theta} = \frac{k}{\theta^2} \frac{1}{\Phi(-k)} > 0$ , where  $\Phi(\cdot)$  denotes the c.d.f. of the standard normal distribution. Part (iii) follows from the fact that  $\lim_{\theta \rightarrow \infty} k = \infty$  and  $\lim_{\theta \rightarrow 0} k = 0$ . The former limit is easy to verify given Lemma 1. The latter limit is apparent from the fact that the  $RHS(k) \equiv \frac{k}{\theta}$  of equation (2) approaches the vertical axis as  $\theta \rightarrow 0$ , therefore if  $\lim_{\theta \rightarrow 0} k$  exists it must be zero. In fact, I established that  $k > 0$  and  $\frac{\partial k}{\partial \theta} > 0$ . Hence  $k$  is an increasing function of  $\theta$  over  $(0, \infty)$  that is bounded below by zero, therefore  $\lim_{\theta \rightarrow 0^+} k$  must exist. By the same logic one can establish that  $\lim_{\theta \rightarrow 0^-} k$  must also exist. The continuity of  $k$  then follows by contradiction because  $\lim_{\theta \rightarrow 0^+} k \neq \lim_{\theta \rightarrow 0^-} k$  would contradict the fact that the function  $RHS(k) = \phi(x) - k\Phi(-k)$  is a continuous function over  $\mathbb{R}$ . ■

**Proof of Proposition 2.** (i) To show that an increase in  $r$  induces a mean preserving spread in the posterior distribution of earnings, one only needs to show that an increase in  $r$  induces a second order stochastic increase in  $\tilde{x}|_{\tilde{r}=r}$  when  $r \in [c, \infty)$  because we already know from Lemma 1, that in equilibrium  $E[\tilde{x}|\tilde{r} = r]$  is constant over

$[c, \infty]$ . Define  $A(y, r) \equiv \int_{-\infty}^y \Pr[\tilde{x} < u|r] du$ , which after some algebra yields  $A(y, r) = \int_{-\infty}^y \{\gamma(r) \Pr[\tilde{x} < u|r, s] + (1 - \gamma(r)) \Pr[\tilde{x} < u|r, w]\} du$  where  $\gamma(r)$  is defined by Corollary 2. There are two cases:  $y \leq r$  and  $y > r$ . The former case is trivial, so I only study the latter. To make the proof more transparent I normalize  $\tilde{x} \sim N(0, 1)$ . After integration by parts,  $A(y, r) \equiv (1 - \gamma(r)) \int_{-\infty}^y \Phi\left(\frac{u}{\sigma}\right) du + \gamma(r)(y - r)$ . Differentiating,  $\frac{\partial A(y, r)}{\partial r} = \frac{\gamma(r)}{r} \left[ \int_{-\infty}^y \Phi(u) du - y \right]$ , where I used the fact that  $\frac{\partial \gamma(r)}{\partial r} = -\frac{\gamma(r)}{r}$ . So I need to show that  $\int_{-\infty}^y \Phi(u) du - y > 0$ . It is immediate to verify that  $\int_{-\infty}^y \Phi(u) du - y$  is a decreasing function of  $y$  and  $\lim_{y \rightarrow \infty} \int_{-\infty}^y \Phi(u) du - y = 0$ , hence the result. (ii) For simplicity I assume  $\tilde{x} \sim N(0, \sigma)$ . First note that  $\text{var}(\tilde{x}|r) = E(\tilde{x}^2|r) - c^2$  because  $E(\tilde{x}|r) = c$  over  $[c, \infty]$ . Furthermore,  $E(\tilde{x}^2|r) = \gamma(r)r^2 + (1 - \gamma(r))\sigma^2$ . Thus  $\text{var}(\tilde{x}|r) = \gamma(r)r^2 + (1 - \gamma(r))\sigma^2 - c^2$ . In equilibrium  $\gamma(r) = \frac{c}{r}$ , thus  $\text{var}(\tilde{x}|r) = cr - \frac{c\sigma^2}{r} + \sigma^2 - c^2$  which, given that  $c > 0$ , implies  $\lim_{r \rightarrow \infty} \text{var}(\tilde{x}|r) = \infty$ . ■

**Proof of Corollary 3.** I first show that  $\text{var}(\tilde{x}|\tilde{r} = r)$  is an increasing function of  $r$ , at a rate that increases in  $\theta$ . It is routine to check  $\frac{\partial}{\partial \theta} \frac{\partial \text{var}(\tilde{x}|\tilde{r} = r)}{\partial r} = \frac{\partial c}{\partial \theta} \left(1 + \frac{\sigma^2}{r^2}\right) > 0$  (for all  $r \geq c$  and zero otherwise), where the inequality holds because  $c$  is an increasing function of  $\theta$  and  $c > 0$ . ■

**Proof of Corollary 4.** This is an implication of integral precision. Let  $\theta > \theta'$  be associated with reports  $r_\theta$  and  $r_{\theta'}$ . By the same arguments given in the proof of Corollary 1,  $r_\theta$  is more integral precise than  $r_{\theta'}$  and therefore  $P(r_\theta) = E[\tilde{x}|r_\theta]$  is greater than  $P(r_{\theta'})$  in the convex order. Given that the variance is defined as the expectation of a convex function, the result follows from the definition of the convex order. ■

**Proof of Corollary 5.** We assume  $\tilde{x} \sim N(0, 1)$ . Now  $\text{var}(\tilde{r}) = E(\tilde{r}^2) - E(\tilde{r})^2$ . But  $E(\tilde{r}) = \gamma * 0 + (1 - \gamma)E[\tilde{r}|w]$ . And  $E[\tilde{r}|w] = \int_c^\infty r\phi(r) dr = \int_c^\infty r\theta \frac{r-c}{c} \phi(r) dr$ . Therefore  $E(\tilde{r}|w) = \frac{\theta}{c} \int_c^\infty (r^2 - rc)\phi(r) dr$ . But  $\int_c^\infty r^2\phi(r) dr = \Phi(-c) + c\phi(c)$  and  $\int_c^\infty cr\phi(r) dr = c\phi(c)$  (see e.g. Tallis, 1971 pp. 224-225). Therefore,  $E[\tilde{r}] = \frac{1}{1+\theta} \frac{\theta}{c} \Phi(-c)$ . But in equilibrium,  $\phi(c) - c\Phi(-c) = \frac{c}{\theta}$ , therefore  $E[\tilde{r}] = \frac{1}{1+\theta} \frac{\Phi(-c)}{\phi(c) - c\Phi(-c)}$ . Similarly,  $E(\tilde{r}^2) = \gamma * 1 + (1 - \gamma)E(\tilde{r}^2|w)$ . Now  $E(\tilde{r}^2|w) = \frac{\theta}{c} \int_c^\infty (r^3 - cr^2)\phi(r) dr$ . But  $\int_c^\infty r^3\phi(r) dr = \phi(c)(2 + c^2)$ . Therefore  $E(\tilde{r}^2|w) = \frac{2\phi(c) - c\Phi(-c)}{\phi(c) - c\Phi(-c)} = 1 + \frac{\phi(c)}{\phi(c) - c\Phi(-c)}$ . Thus,  $E(\tilde{r}^2) = 1 + \frac{1}{1+\theta} \frac{\phi(c)}{\phi(c) - c\Phi(-c)}$ . Now given that  $\frac{1}{1+\theta} = \frac{\phi(c) - c\Phi(-c)}{\phi(c) + c\Phi(c)}$ , we arrive at  $\text{var}(\tilde{r}) = 1 + \frac{\phi(c)}{\phi(c) + c\Phi(c)} - \left(\frac{\Phi(-c)}{\phi(c) + c\Phi(c)}\right)^2$ . Exploiting the strict monotonicity between  $c$  and  $\theta$ , the non monotone relation between  $\text{var}(\tilde{r})$  and  $\theta$  would be established if one shows that  $1 + \frac{\phi(c)}{\phi(c) + c\Phi(c)} - \left(\frac{\Phi(-c)}{\phi(c) + c\Phi(c)}\right)^2$  is non monotone in  $c$  which is immediate to verify (e.g., if



$c = 0.1$ ,  $\text{var}(\tilde{r}) = 0.3$ , if  $c = 1$ ,  $\text{var}(\tilde{r}) = 1.2$  and if  $c = 5$ ,  $\text{var}(\tilde{r}) = 1$ ). ■

**Proof of Lemma 1.** The result follows from differentiating  $\frac{\varphi(r;c)}{\varphi(r;c')} = \frac{\theta \frac{r-c}{c-\mu} f(r)}{\theta' \frac{r-c'}{c'-\mu'} f(r)} = \frac{\theta \frac{r-c}{c-\mu}}{\theta' \frac{r-c'}{c'-\mu'}} = \frac{\theta}{\theta'} \frac{c'-\mu'}{c-\mu} \frac{r-c}{r-c'}$ . Since  $\frac{\theta}{\theta'} \frac{c'-\mu'}{c-\mu}$  is a positive constant, then  $\frac{\partial}{\partial r} \frac{\varphi(r;c)}{\varphi(r;c')} \propto \frac{\partial}{\partial r} \frac{r-c}{r-c'} \propto (r-c') - (r-c) = c-c'$  which is positive by hypothesis. ■

**Proof of Proposition 3.** Some algebra allows us to write express  $EMF = \frac{\sigma}{\delta(k)-k}$  where  $\delta(k) = \frac{\phi(k)}{1-\Phi(k)}$ . Exploiting the monotone relation between  $k$  and  $\theta$ , the result follows from the fact that  $\delta(k) - k$  is a positive and decreasing function of  $k$ ,  $\delta(k) > k$  for all  $k$  and  $\frac{\partial \delta(k)}{\partial k} \in (0, 1)$  (see Heckman and Honoré 1992 pp. 1130). ■

**Proof of Corollary 6.** Let  $c_\gamma$  be the kink in the distribution of  $\tilde{r}$  when the quality of the ICS is  $\gamma$ . Let  $g_\gamma(r) \equiv G'_\gamma(r)$ .

$$g_\gamma(r) = \begin{cases} \gamma f(r) & \text{if } r < c_\gamma \\ \gamma \frac{r}{c_\gamma} f(r) & \text{if } r \geq c_\gamma \end{cases} \quad (26)$$

Note that  $\gamma \frac{r}{c_\gamma} f(r)$  is zero at  $r = 0$  and attains a maximum at  $r = \sigma$ , regardless of  $\gamma$ . Assume that  $\gamma > \gamma'$ . I will show that  $g_\gamma(r)$  single cross  $g_{\gamma'}(r)$  from above, which implies FOSD. Define  $\Delta \equiv G_\gamma - G_{\gamma'}$ .  $\Delta$  is positive for  $r < c_{\gamma'}$  and negative for  $r > c_\gamma$  (given that  $\frac{\gamma}{c_\gamma}$  decreases in  $\gamma$ ). Thus  $\Delta$  must cross zero over the interval  $[c_{\gamma'}, c_\gamma]$ . We show that  $\Delta$  crosses zero only once over  $[c_{\gamma'}, c_\gamma]$ . Suppose not, then  $\Delta' = 0$  at least twice over  $r \in [c_{\gamma'}, c_\gamma]$ . But,  $\Delta = \gamma f(r) - \frac{\gamma'}{c_{\gamma'}} r f(r)$  so that  $\Delta' = f(r) \left\{ -\gamma r - \frac{\gamma'}{c_{\gamma'}} + \frac{\gamma'}{c_{\gamma'}} r^2 \right\}$ . Thus  $\Delta' = 0 \iff r \in \left\{ \frac{\left( \gamma + \sqrt{\gamma^2 + 4 \left( \frac{\gamma'}{c_{\gamma'}} \right)^2} \right)}{2 \frac{\gamma'}{c_{\gamma'}}}, \frac{\left( \gamma - \sqrt{\gamma^2 + 4 \left( \frac{\gamma'}{c_{\gamma'}} \right)^2} \right)}{2 \frac{\gamma'}{c_{\gamma'}}} \right\}$ . But  $\left( \gamma - \sqrt{\gamma^2 + 4 \left( \frac{\gamma'}{c_{\gamma'}} \right)^2} \right) < 0 < c_{\gamma'}$  which is a contradiction. ■

I will prove Lemma 2 Proposition 4 in a general way by assuming that the report  $\mathbf{r}_t = \{r_1^t, r_2^t, \dots, r_t^t\}$  is  $t$  dimensional. In this context  $r_j^t$  is period  $j$  earnings as reported in period  $t$ . The equilibrium reporting strategy  $\hat{\varphi}_t(\cdot | h_t)$  will represent here a joint p.d.f. that describes the probability density the manager assigns to reporting  $\mathbf{r}_t$  in equilibrium, given a history  $h_t$ .

**Proof of Lemma 2.** Define the set  $A_t(B) \equiv \left\{ \mathbf{r}_t : \sum_{j=1}^t r_j^t = B \right\}$  the set of reports that induce a book value  $B$ . Pick any two reports  $\mathbf{r}_t, \mathbf{r}'_t \in A_t(B)$ . Case (i): suppose that both  $\mathbf{r}_t, \mathbf{r}'_t$  belong to the support of  $\hat{\varphi}_t(\cdot | h_t)$ . Then, the reports  $\mathbf{r}_t$  and  $\mathbf{r}'_t$  must induce the same constant price in equilibrium, thus  $E[X | \mathbf{r}_t, h_t] = E[X | \mathbf{r}'_t, h_t]$ . Case (ii): sup-

pose that neither of  $\mathbf{r}_t$  nor  $\mathbf{r}'_t$  belong to the support of  $\hat{\varphi}_t(\cdot|h_t)$ . Then since report  $\mathbf{r}_t$  is not in  $\varphi(\cdot)$ 's support, it must be that for the report  $\mathbf{r}_t$  to occur, the ICS is effective. So, since  $B = \sum_{k=1}^t r_k^t$ ,  $B$  must be the firm's value. The same is also true for any other  $\mathbf{r}'_t \notin \text{support} \hat{\varphi}_t(\cdot|h_t)$ . Finally we need to show that for any pair of reports  $\mathbf{r}_t, \mathbf{r}'_t \in A_t(B)$ , either they both belong to the support of  $\hat{\varphi}_t(\cdot|h_t)$  or none of them belong to the support of  $\hat{\varphi}_t(\cdot|h_t)$ . Suppose not, so that only  $\mathbf{r}_t \in \text{support} \hat{\varphi}_t(\cdot|h_t)$  but  $\mathbf{r}'_t \notin \text{support} \hat{\varphi}_t(\cdot|h_t)$ . Clearly  $E(X|\mathbf{r}'_t, h_t) = B_t$ . But I claim that  $E(X|\mathbf{r}_t, h_t) < B_t = E(X|\mathbf{r}'_t, h_t)$  which contradicts the fact that  $\mathbf{r}_t$  is an optimal report under  $\tau_t = w$ , namely a report that belongs to the support  $\hat{\varphi}_t(\cdot|h_t)$ . To prove that  $E(X|\mathbf{r}_t, h_t) < B_t$  we proceed by induction. We know that  $p_1 = E(X|r_1) < B_1 = r_1$  by Lemma 1. Assume that  $p_{t-1} \leq B_{t-1}$ . Then,  $P_t = \gamma_t B_t + (1 - \gamma_t) p_{t-1} \leq \gamma_t (B_{t-1} + r_t) + (1 - \gamma_t) B_{t-1} = B_{t-1} + \gamma_t r_t \leq B_t$  with the last inequality being strict whenever  $\gamma_t > 0$ . ■

**Proof of Proposition 4.** The proof is given in three steps: first I take advantage of Lemma 2 to express the probability  $\Pr\{\tau_t = w|\mathbf{r}_t, w\}$  as a function of the reported increment  $r_t$  rather than the overall report  $\mathbf{r}_t$ . Second, using the same arguments invoked in the proof of Lemma 1, I look for the equilibrium reporting strategy  $\varphi_t(\cdot|h_t)$  associated to the increment  $r_t$ , with support on an interval like  $[c_t, \infty)$ . Finally, I show that there exists a unique  $c_t$  that satisfies the requirements of an equilibrium. Using Bayes rule

$$\begin{aligned} \gamma_t(\mathbf{r}_t, h_t) &= \frac{\Pr(\mathbf{r}_t, h_t|\tau_t = s) \gamma}{\Pr(\mathbf{r}_t, h_t)} = \frac{\Pr(\mathbf{r}_t, h_t|\tau_t = s) \gamma}{\Pr(\mathbf{r}_t|h_t) \Pr(h_t)} \\ &= \frac{\Pr(\mathbf{r}_t|h_t, \tau_t = s) \Pr(h_t|\tau_t = s) \gamma}{\Pr(\mathbf{r}_t|h_t) \Pr(h_t)} \\ &= \frac{\Pr(\mathbf{r}_t|h_t, \tau_t = s) \gamma}{\Pr(\mathbf{r}_t|h_t)} = \frac{\Pr(\mathbf{r}_t|h_t, \tau_t = s) \gamma}{\Pr(\mathbf{r}_t|h_t)}. \end{aligned} \quad (27)$$

By Lemma 2,

$$\gamma_t(\mathbf{r}_t, h_t) = \frac{\gamma f_t(r_t|h_t)}{\gamma f_t(r_t|h_t) + (1 - \gamma) \varphi_t(r_t|h_t)}. \quad (28)$$

The value of  $\gamma_t = \gamma_t(\mathbf{r}_t, h_t)$  characterizes the credibility of all reports that induce the same book value  $B_t$  (or the same increment  $r_t$ ) as  $\mathbf{r}_t$  does. Consider now how the price is determined. By the law of iterated expectations, the pricing function follows

$$\begin{aligned} P_t &= \gamma_t B_t + (1 - \gamma_t) p_{t-1} \\ &= \gamma_t (B_{t-1} + r_t) + (1 - \gamma_t) p_{t-1}. \end{aligned} \quad (29)$$

That is, with probability  $\gamma_t$  the firm's ICS is  $\tau_t = s$  implying that the reported book value  $B_t$  is true. With probability  $1 - \gamma_t$  the firm's ICS is weak, therefore the report is uninformative and the current price is not updated but set at its prior level  $p_{t-1}$ . Now, we note that the pricing function must be flat for all reported increments  $r_t$  in the support of  $\varphi_t(\cdot|h_t)$ . So if support  $\varphi_t(\cdot|h_t)$  is defined as an interval like  $[c_t, \infty)$  then the pricing function must obey

$$P_t = B_{t-1} + \min(r_t, c_t). \quad (30)$$

Note that the intercept of the pricing function must be  $B_{t-1}$  if the pricing function is to be continuous at  $c_t$ . Combining Eq. (29) and Eq. (30), we obtain  $\gamma_t B_t + (1 - \gamma_t) p_{t-1} = B_{t-1} + c_t$  over  $[c_t, \infty)$ . Now substituting  $\frac{\gamma f_t(r_t|h_t)}{\gamma f_t(r_t|h_t) + (1-\gamma)\varphi_t(r_t|h_t)}$  and solving for  $\varphi_t(\cdot|h_t)$  we arrive at

$$\varphi(r_t|h_t) = \begin{cases} \theta \frac{r_t - c_t}{B_{t-1} - p_{t-1} + c_t} f_t(r_t|h_t) & \text{if } r_t < c_t \\ 0 & \text{if } r_t \geq c_t \end{cases}. \quad (31)$$

By the same arguments given in Lemma 1, there is a unique  $c_t$  such that  $\int_{c_t}^{\infty} \varphi(r_t|h_t) dr_t = 1$  and  $\varphi(r_t|h_t) \geq 0$  for all  $r_t$ . ■

**Proof of Corollary 8.** Note that  $E(\tilde{r}_t|h_t) = (1 - \gamma) E(\tilde{r}_t|h_t, \tau_t = w)$ . Consider two histories,  $h_t$  and  $h'_t$  leading to  $B_{t-1}, B'_{t-1}$  such that  $B_{t-1} > B'_{t-1}$ , then, by the same arguments provided in the proof to Corollary 7, it is easy to verify that  $c'_t > c_t$  where  $c_t$  is the kink associated with the history  $h_t$  and  $c'_t$  is the kink associated with the history  $h'_t$ . On the other hand, if  $c'_t > c_t$  the ratio  $\frac{\sigma_t(r_t|h_t)}{\sigma_t(r_t|h'_t)}$  decreases in  $r_t$  thus the family of distributions  $\{\sigma_t(\cdot|h_t(B_{t-1}))\}_{B_{t-1}}$  satisfies MLRP which is sufficient to ensure that  $E(\tilde{r}_t|h_t, \tau_t = w)$  decreases in  $B_{t-1}$ . ■

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